Math 61-1 Midterm 2 version b

TOTAL POINTS

47 / 50

QUESTION 1

Multiple choice 10 pts

1.1 **2 / 2**

✓ **- 0 pts** **Correct (d)**

- 2 pts Incorrect

1.2 **2 / 2**

✓ **- 0 pts** **Correct (d)**

- 2 pts Incorrect

- 2 pts No Answer

1.3 **2 / 2**

✓ **- 0 pts** **Correct (b)**

- 2 pts Incorrect

1.4 **2 / 2**

✓ **- 0 pts** **Correct (c)**

- 2 pts Incorrect

1.5 **2 / 2**

✓ **- 0 pts** **Correct (b)**

- 2 pts Incorrect

QUESTION 2

Short answer 10 pts

2.1 **2 / 2**

✓ **- 0 pts** **Correct (960)**

- 2 pts Incorrect

- 0.5 pts Gave term, not just coefficient

- 1.5 pts Didn't multiply by 8 (120) or similar

2.2 **2 / 2**

✓ **- 0 pts** **Correct (34650)**

- 2 pts Incorrect

2.3 **1 / 2**

- 0 pts Correct (302400)

- 2 pts Incorrect

- 2 pts No answer

- 1 **Point adjustment**

Close

2.4 **0 / 2**

- 0 pts Correct (21)

✓ **- 2 pts** **Incorrect**

- 2 pts No answer

2.5 **2 / 2**

✓ **- 0 pts** **Correct (176)**

- 2 pts Incorrect

- 2 pts No answer

QUESTION 3

Euler paths 10 pts

3.1 criteria for euler paths **5 / 5**

✓ **+ 5 pts** **Correct**

+ 2 pts Euler cycle criterion

+ 1 pts Euler cycle criterion (missing connected or mistake)

+ 2 pts Reduction to even degree graph

+ 1 pts Reduction to even degree graph (with mistake)

+ 1 pts Correct explanation of how to get Euler path from Euler cycle

+ 0 pts Incorrect

3.2 application of euler paths **6 / 5**

✓ **- 0 pts** **Correct**

+ 2 pts Click here to replace this description.

✓ + 1 pts **Click here to replace this description.**

QUESTION 4

4 pigeon hole 10 / 10

✓ - 0 pts **Correct**

- 3 pts Wrong partition
- 3 pts No Pigeonhole
- 2 pts Minor error
- 10 pts Blank

QUESTION 5

hypercube 10 pts

5.1 recurrence for edges 4 / 4

✓ - 0 pts **Correct**

- 4 pts empty
- 1 pts Sounds like you are saying the n-1 dim hypercube has n-1 vertices
- 1 pts need to explain how the hypercube is built out of smaller ones
- 4 pts empty
- 0.5 pts How do you get the recurrence?
- 2 pts large mistake
- 2 pts you are assuming the desired conclusion
- 3 pts can't just do examples

5.2 formula for edges 5 / 6

- 0 pts Correct
- 4 pts wrong answer, need to use iteration

✓ - 1 pts **need to use induction to show that answer is true**

- 3 pts wrong answer, this is why you need to use induction to show answer is true
- 6 pts empty
- 4 pts incomplete
- 2 pts wrong answer

Midterm 2

Name: ______________________________

Student ID: ______________________________

Section:

	Tuesday:	Thursday:	
	1A	1B	TA: Albert Zheng
	1C	1D	TA: Benjamin Spitz
	1E	1F	TA: Eilon Reisin-Tzur

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators,** books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper, nor may you work on the exam after it has ended. There is some scratch paper at the back of the exam, if you do work there indicate so on the relevant question. Do not write on the backs of the pages. Please circle or box your final answers.

Please get out your id and be ready to show it when you turn in your exam.

Please do not write below this line.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Circle the correct answer (only one answer is correct for each question)

1. If X is a set with 10 elements and Y is a set with 3 elements and $f : X \to Y$:
   (a) f could be one-to-one
   (b) it is possible that there are *not* 4 distinct elements of X, x_1, x_2, x_3, x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4)$.
   (c) there are at least 5 distinct elements of X, x_1, x_2, x_3, x_4, x_5 with $f(x_1) = f(x_2) = f(x_3) = f(x_4) = f(x_5)$.
   (d) there are at least 4 distinct elements of X, x_1, x_2, x_3, x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4)$.

2. The number of relations on a set with n elements that are both symmetric and reflexive is:
   (a) $2^{n^2} + 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$
   (b) $2^{n^2-n} + 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$
   (c) 2^{n^2}
   (d) none of the above

$(a,1) \to (1,a)$

reflexive:

2^{n^2-n}

both

2

symmetric (not reflexive)

$2^{\frac{n^2}{2}-\frac{n}{2}}$

3. K_n has an Euler cycle:
   (a) For no n
   (b) For n odd
   (c) For n even
   (d) For every n

Question 1 continues on the next page...

Question 1 continued...

4. If $s_0 = 2$ and $s_1 = 1$ and for $n \geq 2$ $s_n = s_{n-1} + s_{n-2}$ then $s_n =$

   (a) $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$

   (b) $\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

   (c) $\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$

   (d) none of the above

5. The number of ways of arranging a math book, a history book, a computer science book, and an English book on a shelf is:

   (a) $4 - 3 + 2 - 1$

   (b) $4!$

   (c) $1 + 2 + 3 + 4$

   (d) 4

2. In this question write down your answer, no need for any justification. Leave your answers in a form involving factorials, $P(n, m)$, $\binom{n}{m}$, exponents, etc.

10 9 8 7
0 1 2 3

(a) (2 points) What is the coefficient of x^7y^3 in the polynomial $(x+2y)^{10}$?

$$\binom{10}{3}(1)^7(2)^3 = 8\binom{10}{3}$$

(b) (2 points) What is the number of ways of rearranging the letters of the word "MISSISSIPPI"? 4S 4I 2P

$$\frac{11!}{4!4!2!}$$

(c) (2 points) I have 10 distinct books. Two are by Ernest Hemingway and three are by Virginia Woolf. How many ways are there to arrange these books on a shelf if the ones by Hemingway must be next to each other and the ones by Woolf must not be next to each other?

$$2!\,6!\binom{7}{3}$$

(d) (2 points) What is the number of binary strings of length 6 that don't contain 11 as a substring?

0 _ 0 _ 0 _ 0 _

choose 0s → choose 2 slots to put ones in

$$\binom{6}{0}+\binom{6}{1}+\binom{4}{4}\binom{5}{2}$$

(e) (2 points) What is the number of solutions to the equation $x_1+x_2+x_3 = 20$ where $x_3 \leq 10$ and x_1, x_2, x_3 are nonnegative integers?

$$\binom{21}{1}+\binom{20}{1}+\binom{19}{1}+\binom{18}{1}+\binom{17}{1}+\binom{16}{1}+\binom{15}{1}+\binom{14}{1}+\binom{13}{1}+\binom{12}{1}+\binom{11}{1}$$

3. (a) (5 points) Show that if a connected graph has exactly 2 vertices of odd degree, then there is a path in the graph that visits each edge exactly once. (Feel free to use what we know about Euler cycles from lecture)

let the two vertices w/ odd degree be v_1 and v_2

construct a graph G' by inserting an edge e between v_1 and v_2

now all vertices have even degree and the graph is connected, so there is an Euler cycle at v_1

$P = v_1 \dots v_2\, e\, v_1$. now, if we replace $v_2\, e\, v_1$ with v_2, we have a path that visits each edge exactly once from v_1 to v_2

Question 3 continues on the next page...

(b) (5 points) A house has a front door and a back door and no other doors leading outside. Inside each room of the house, there are (exactly) two doors that each go to other rooms of the house [1] (and the only way to enter or leave a room is through a door). Each room can be reached from the front door. Show that it is possible to go in through the front door and leave through the back door while going through each door in the house exactly once. Feel free to use the previous part of this problem.

let each room ~~including~~ the outside front and outside back be a vertex in a graph G, and an edge denote a door between two rooms (including outside). now G is connected and all vertices have even degree except for two.

by the previous part of the problem, it is possible to go through each door (each edge) exactly once, starting with the front door and ending with the back door (edges incident on the vertices w/ odd edges)

[1]To clarify the rooms just inside the front and the back doors have three doors, one of them is the front/back door, and the other two are the doorways leading to other rooms of the house

4. (10 points) Suppose that six distinct integers are selected from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Prove that at least two of these six integers sum to 11.

consider the partition

$\{\{1,10\}, \{2,9\}, \{3,8\}, \{4,7\}, \{5,6\}\}$, where each element is a set of two numbers whose sum is 11. now, if we pick 6 distinct integers from the original set, these are our pigeons and the elements of the partition are the holes.

by pigeonhole principle, 2 numbers will be in the same element of our partition, thus at least 2 of the 6 integers sum to 11.

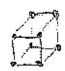
5. Recall the n-dimensional hypercube. This is a graph with vertices labeled by binary strings of length n, with an edge between two vertices if they differ in exactly one digit. Let c_n be the number of edges in the n-dimensional hypercube.

(a) (4 points) Show that c_n satisfies the recurrence $c_n = 2^{n-1} + 2c_{n-1}$

$c_0 = 0$

$c_1 = 1$

$c_2 = 4$ 

$c_3 = 12$ 

$c_4 = 2^3 + 24 = 32$

consider c_{n-1} edges in an $(n-1)$ dimensional hyper cube. if we construct an n dimensional hyper cube, we have $2c_{n-1}$ edges from the two distinct $(n-1)$ D hyper cubes. now, there are 2^{n-1} pairs of vertices (one from each $(n-1)$ D hyper cube) such that the cubes differ by exactly one digit.

so the total edges in an n-D hypercube is $c_n = 2^{n-1} + 2c_n$

Question 5 continued...

(b) (6 points) Solve your recurrence from the previous part of this question to find a formula for c_n.

$C_n = 2^{n-1} + 2c_{n-1}$

$C_n = 2^{n-1} + 2(2^{n-2} + 2C_{n-2})$

$C_n = 2^{n-1} + 2^{n-1} + 2 \cdot 2 \cdot C_{n-2}$

$= 2^{n-1} + 2^{n-1} + 2 \cdot 2 \cdot (2^{n-3} + 2C_{n-3})$

$= 3 \cdot 2^{n-1} + 2^3 C_{n-3}$

$C_n = k \cdot 2^{n-1} + 2^k C_{n-k}$

$C_1 = 1$ so let $k = n-1$

$n - k = 1$

$k = n-1$

$C_n = (n-1)2^{n-1} + 2^{n-1} C_1$

$\boxed{C_n = n 2^{n-1}}$

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