Midterm 1

Last Name: Miller

First Name: Daniel

Student ID: 004786138

Section: Tuesday: Thursday:

2A 2B TA: Alex Mennen

2C 2D TA: Van Latimer

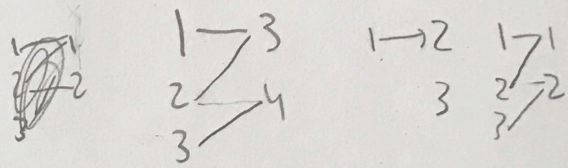
Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators,** books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it.
Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper. If you write on the exam before the exam starts or after it end, this will be considered and act of academic dishonesty.
You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. Please circle or box your final answers.
Please get out your id and be ready to show it when you turn in your exam.

Please do not write below this line.

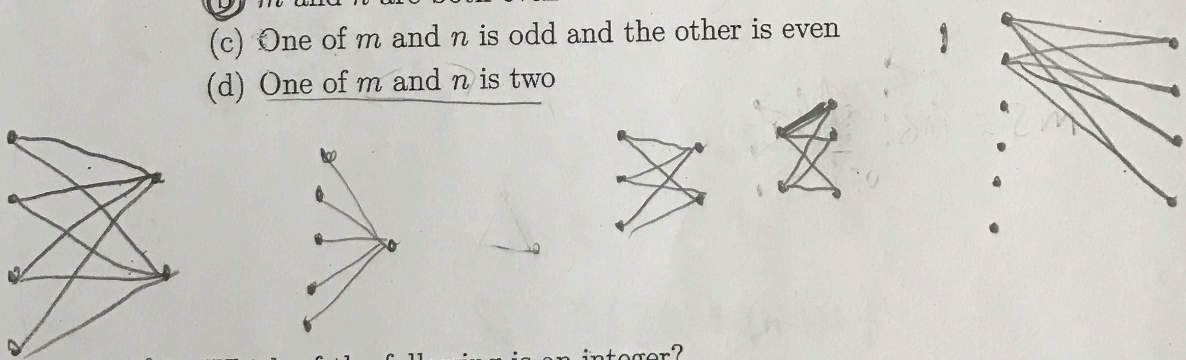
Question	Points	Score
1	10	6
2	10	10
3	10	5
4	10	10
5	10	5
Total:	50	36

1. (10 points) The follow questions have one correct answer, indicate which answer is correct.

   1. If X and Y are finite sets and every function from X to Y is not injective, then:
      (a) $|X| < |Y|$
      (b) $|X| > |Y|$
      (c) $|X| = |Y|$
      (d) $|X|$ could be any of the following: larger than, smaller than, or equal to $|Y|$



   2. The complete bipartite graph $K_{m,n}$ has an Euler cycle when:
      ~~(a) m and n are both odd~~
      (b) m and n are both even
      (c) One of m and n is odd and the other is even
      (d) One of m and n is two



   3. Which of the following is an integer?
      (a) $\left(\frac{1+\sqrt{5}}{2}\right)^{100} - \left(\frac{1-\sqrt{5}}{2}\right)^{100}$
      (b) $\left(\frac{1+\sqrt{5}}{2}\right)^{100} + \left(\frac{1-\sqrt{5}}{2}\right)^{100}$
      (c) $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{100} + \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{100}$
      (d) $\left(\frac{1+\sqrt{5}}{2}\right)^{100}$

Question 1 continues on the next page. . .

4. Consider the sequence with first term a_0 defined for $n \geq 3$ by the recurrence relation $a_n = a_{n-1} + 2a_{n-3}$. If $a_1 = 1$, $a_2 = 4$, and $a_3 = 2$, what is a_0?

a_3 $a_n =$

(a) $a_0 = 1$

(b) $a_0 = 0$

$-1, 4, 2$

(c) $a_0 = -1$

$2 = 4 +$

(d) Not enough information is given to determine the answer.

$a_0 = 3$

5. What is the sum of the degrees of the vertices in K_n?

(a) $n(n+1)$

(b) $(n-1)n$

(c) $\frac{(n-1)n}{2}$

(d) $\frac{n(n+1)}{2}$

$\sum_{i=1}^{n}$

$1, 2, 3, 4, 5 .. n$

$d = 1$

$S_n = \frac{n}{2}(2(1) + (n-1)1)$

$\frac{n(n+1)}{2}$

2. (10 points) Find a formula for the the recurrence relation $a_n = -2a_{n-1} + 8a_{n-2}$ with initial conditions $a_0 = 2$, $a_1 = -2$.

$a_n = -2a_{n-1} + 8a_{n-2}$

$a_n + 2a_{n-1} - 8a_{n-2} = 0$

$t^2 + 2t - 8 = 0$ ✓

$(t+4)(t-2)$

$t = -4, 2$ ✓

$\alpha(-4)^n + \beta(2)^n$ ✓

$2(2 = \alpha + \beta)$ ✓

$-2 = -4\alpha + 2\beta$ ✓

$4 = 2\alpha + 2\beta$

$-\ -2 = -4\alpha + 2\alpha$

$6 = 6\alpha$

$\alpha = 1$

$\beta = 1$

$\boxed{a_n = (-4)^n + (2)^n}$ ✓

3. An *Euler path* in a graph is a path (not necessarily a cycle) that visits each edge in the graph exactly one time.

(a) (7 points) Show that a connected graph G has an Euler path if and only if either every vertex in G has even degree, or G has exactly two vertices of odd degree.

Going in First direction,

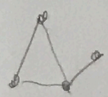
If G has ~~odd~~ even degree, this means there is an Euler cycle. ~~And~~ Given an Euler path visits each edge once, which is one of the properties of an Euler cycle, this would mean there is an Euler path.

(+2)

No.

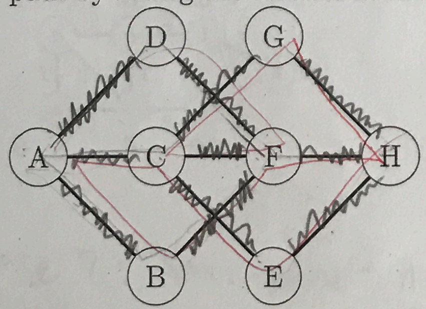
Second direction

Very hard to follow



If there is an Euler path, every edge must have been visited once. ~~Therefore this~~ Essentially, this works because by adding two of odd degree it is like adding two edges to the graph. This would in fact keep the graphs sum of edges even and would thus allow for an Euler cycle, that would allow an Euler path as explained before. ~~I~~If we had more than two, even if even, different ~~per~~ arrangements would not work as we could put more than ~~three~~ three on one vertex's degree.

(b) (3 points) Find an Euler path in the following graph. Decribe your path by listing the vertices in the order that they appear in the path.

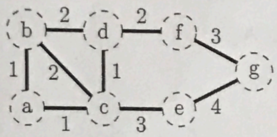


A, D, F, C, G, H, F, B, A, C, E, H

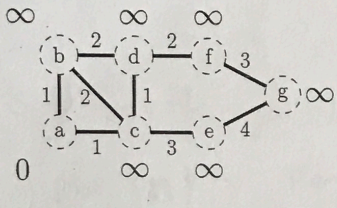
(3)

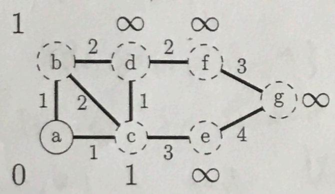
5

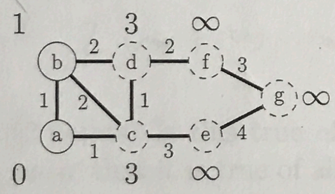
4. (10 points) I am using Dijkstra's algorithm to find the length of the shortest path from vertex a to vertex g in the following weighted graph. When I circle a vertex, I fill in the dotted circles around each vertex.

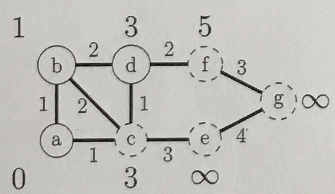


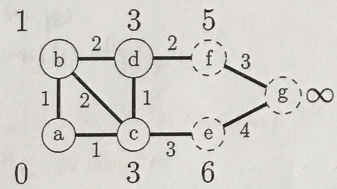
At what stage do I make an error, and what is my error?

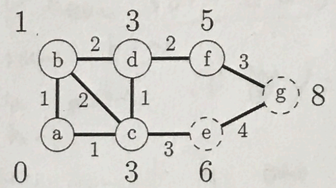
1. 

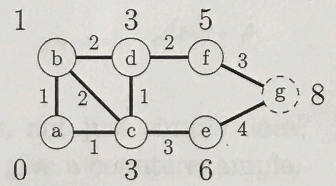
2. 

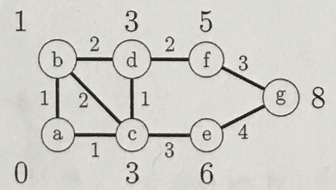
3. 

4. 

5. 

6. 

7. 

8. 

Stage 2: ~~Your first~~ After circling b, you add 2+1 for vertex c and put a 3. This should not be done as you only change the value on top of the vertex if it is less than the current one. 3 is not less than 1, so this should NOT be done.

5. (a) (4 points) Show that in any simple graph with two or more vertices, there must be at least two vertices that have the same degree.

2 vertices

base case  it works, what if

2/4

Simple graph: undirected
un weighted
no loops
no parallel edges

This is totally the right idea but what is the relationship between this paragraph and the problem

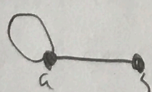
n pigeons {n} cannot have both 0 and n ?

n-1 holes {~~0~~,1,2,3..n} at same time

Hence by second pigeonhole princ, if X = pigeons, Y = holes and |X| > |Y|, |n| > n-1, there must be at least 2 vertices with the same degree

(b) (2 points) Is this true of all graphs, not just simple ones? Either prove that it is true of all graphs or give a counterexample.

2/2

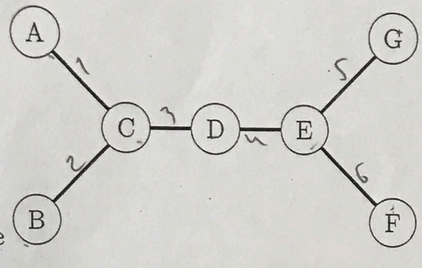


not true, counter example

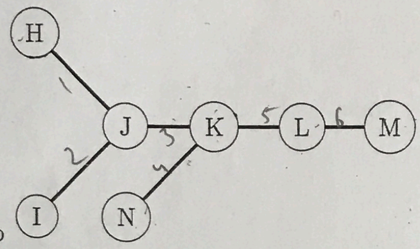
a has degree 3 and b has degree 1.

Question 5 continues on the next page...

Question 5 continued...



Graph One



Graph Two

(c) (4 points) Are the above two graphs isomorphic? Be sure to justify your answer.

1/4

1 2 3 4 5 6 7
A B C D E F G

1 2 3 4 5 6 7
H J I K N L M

6 ~~vertices~~ edges. both

7 vertices both

A) deg 1 H) deg 1

B) deg 1 I) deg 1

C) deg 3 J) deg 3

D) deg 2 L) deg 2

E) deg 3 K) deg 3

G) deg 1 N) deg 1

F) deg 1 M) deg 1

It looks like they are. They have equal amounts of vertices, and edges. Also, they have the same degrees on their corresponding vertices. Hence by the corollary that G_1 vert map to G_2 vert and $f(v)$ must be adjacent to $f(w)$ iff v and w are adjacent, this holds true for all and thus they are isomorphic.