# 21W-MATH61-1 Exam One

ERIC YANG

TOTAL POINTS

**37 / 40**

QUESTION 1

## Short answer 10 pts

**1.1 0 / 2**

- **- 0 pts** Correct
- **- 1 pts** Right idea with a computation mistake, or slightly off number
- ✓ **- 2 pts** 0

**1.2 2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** You listed the classes, not an element from each class.
- **- 2 pts** incorrect
- **- 1 pts** Right idea, but incorrect final answer

**1.3 2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** incorrect

**1.4 2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** incorrect
- **- 1 pts** Right idea or close answer

**1.5 2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** incorrect
- **- 1 pts** Ambiguous answer.

QUESTION 2

## Even and odd sets 10 pts

**2.1 2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** Missing some sets
- **- 2 pts** blank/ several mistakes
- **- 1 pts** should be the empty set, not the set containing the empty set

**2.2 8 / 8**

- ✓ **- 0 pts** Correct

Proof with induction

- **- 1 pts** issue with base case
- **- 2 pts** many issues with inductive hypothesis
- **- 2 pts** no base case/ several issues in base case
- **- 1 pts** issue with inductive hypothesis
- **- 1 pts** mistake in logic of inductive step
- **- 2 pts** many issues in logic of inductive step
- **- 1 pts** mistake in doing inductive step
- **- 2 pts** many issues in doing inductive step

Counting proof

- **- 4 pts** fails to construct bijection between 0(X) and E(X)
- **- 2 pts** fails to show why |O(X)|=|E(X)|
- **- 3 pts** many issues with bijection between O(X) and E(X)

Using binomial theorem

- **- 4 pts** doesn't explain binomial theorem
- **- 2 pts** doesn't have a clear/ correct statement for the binomial theorem
- **- 2 pts** doesn't apply binomial theorem correctly/usefully

QUESTION 3

10 pts

**3.1 3 / 4**

$$\mathcal{E}$$

- **- 0 pts** Correct – $$\{ \{0,2\}, \{1,3\}\}$$ or $$\{ [0],[1] \}$$.

✓ **- 1 pts Listed ordered pairs instead of sets**

- **1 pts** Listed with repetition
- **1 pts** Listed representatives instead of equivalence classes
- **2 pts** Incorrect

$$h$$

✓ **- 0 pts Correct**

- **1 pts** Correct except missing $$([0]_R, b)$$ and/or $$([1]_R, b)$$
- **1 pts** Other – see comment
- **2 pts** Incorrect

Your answer for $$h$$ is incorrect, but would be correct if your answer for $$\mathcal{E}$$ were correct

**3.2 6 / 6**

Definition of $$h$$

✓ **+ 2 pts Correctly used**

+ **1 pts** Uses relation from part (a) instead of a general one
+ **1 pts** Usage not clear/not fully correct
+ **0 pts** Incorrect/not used/didn't explain usage

Overall logic

✓ **+ 2 pts Correct**

+ **1 pts** Right idea but unclear/minor errors
+ **0 pts** Incorrect/missing

Definition of "function" applied to $$h$$

✓ **+ 2 pts Correctly used**

+ **1 pts** Not completely correct/correctly used (e.g. didn't use uniqueness property)
+ **0 pts** Incorrect/not used

1 What are $$A$$ and $$y$$ here?

2 element

3 You've shown that if $$x,y\in A$$ for some $$A\in\mathcal{E}$$, then $$f(x)=f(y)$$. You still need to show (mention) that if $$(x,y)\in R$$, then $$x,y\in A$$ for some $$A\in\mathcal{E}$$.

QUESTION 4

10 pts

**4.1 5 / 5**

✓ **- 0 pts Correct**

- **2 pts** Wrong defintion of onto
- **1 pts** Overall logic
- **2 pts** Wrong definition of composition

**4.2 5 / 5**

✓ **+ 5 pts Correct**

+ **2 pts** Gave/used definition of 1-1
+ **2 pts** Demonstrated/used definition of composition of functions



gradescope

Eric Yang
ID#[illegible]

1a) $2^n$

1b) $\{1, 2, 3, 4\}$

1c) $S = \{(a, c), (b, b), (c, a)\}$ $R = \{(a, b) (b, a), (c, c)\}$

1d) $2^5 = 32$

1e) $X = \{1, 2\}$ $f = \{(1, 2), (2, 1)\}$
$Y = \{1, 2, 3\}$ $g = \{(2, 1), (1, 2), (3, 1)\}$

**1.1 0 / 2**

- **- 0 pts** Correct
- **- 1 pts** Right idea with a computation mistake, or slightly off number
- ✓ **- 2 pts** **0**



gradescope

Eric Yang
ID#205505249

1a) $2^n$

1b) $\{1, 2, 3, 4\}$

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$Y = \{1, 2, 3\}$ $g = \{(2, 1), (1, 2), (3, 1)\}$

**1.2 2 / 2**

✓ **- 0 pts** **Correct**

**- 1 pts** You listed the classes, not an element from each class.

**- 2 pts** incorrect

**- 1 pts** Right idea, but incorrect final answer



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Eric Yang
ID#205505249

FIVE STAR.

1a) $2^n$

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**1.3 2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** incorrect



gradescope

Eric Yang
ID# 205505249

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**1.4 2 / 2**

- ✓ **- 0 pts** Correct
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Eric Yang
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**1.5 2 / 2**

- ✓ **- 0 pts** Correct
- **- 2 pts** incorrect
- **- 1 pts** Ambiguous answer.



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2a) $\mathcal{P}(\{a,b,c\}) = \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$

$\Rightarrow \mathcal{O}(\{a,b,c\}) = \{a\}, \{b\}, \{c\}, \{a,b,c\}$

$\Rightarrow \mathcal{E}(\{a,b,c\}) = \emptyset, \{a,b\}, \{a,c\}, \{b,c\}$

2b) Base case: $|X| = 1$, $\mathcal{O}(X) = X$, $\mathcal{E}(X) = \emptyset \Rightarrow |\mathcal{O}(X)| = |\mathcal{E}(X)| = 2^0 = 1$

Inductive step: Assume for $|X| = n$, $|\mathcal{O}(X)| = |\mathcal{E}(X)| = 2^{n-1}$. Let $|Y| = n+1$ where $Y$ contains $X$ and an extra element called $z$. Thus, $A \in \mathcal{P}(Y)$ : $|A|$ is odd, or $\mathcal{O}(Y)$, contains $\mathcal{O}(X)$ and $\mathcal{E}(X) \cup \{z\}$. This means that $|\mathcal{O}(Y)| = |\mathcal{O}(X)| + |\mathcal{E}(X)| = 2^n$. Using the fact we know that $|\mathcal{P}(Y)| = 2^{|Y|} = 2^{n+1}$, we then know that $|\mathcal{E}(Y)| = 2^n$ since all the elements of $\mathcal{P}(Y)$ not a part of $\mathcal{O}(Y)$ are in $\mathcal{E}(Y)$.

Therefore, we have proved that if $|\mathcal{O}(X)| = |\mathcal{E}(X)| = 2^{|X|-1}$ for $X$, then $|\mathcal{O}(Y)| = |\mathcal{E}(Y)| = 2^{|X|}$ where $|Y| = |X| + 1$.

Using proof by induction, we have thus shown that for $|X|$ finite and $|X| \geq 1$, $|\mathcal{O}(X)| = |\mathcal{E}(X)| = 2^{|X|-1}$.

**2.1 2 / 2**

- ✓ **- 0 pts** Correct
- **- 1 pts** Missing some sets
- **- 2 pts** blank/ several mistakes
- **- 1 pts** should be the empty set, not the set containing the empty set



gradescope

2a) $\mathcal{P}(\{a,b,c\}) = \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$

$\Rightarrow O(\{a,b,c\}) = \{a\}, \{b\}, \{c\}, \{a,b,c\}$

$\Rightarrow \mathcal{E}(\{a,b,c\}) = \emptyset, \{a,b\}, \{a,c\}, \{b,c\}$

2b) Base case: $|X| = 1$, $O(X) = X$, $\mathcal{E}(X) = \emptyset \Rightarrow |O(X)| = |\mathcal{E}(X)| = 2^0 = 1$

Inductive step: Assume for $|X| = n$, $|O(X)| = |\mathcal{E}(X)| = 2^{n-1}$. Let $|Y| = n+1$ where $Y$ contains $X$ and an extra element called $z$. Thus, $A \in \mathcal{P}(Y)$ : $|A|$ is odd, or $O(Y)$, contains $O(X)$ and $\mathcal{E}(X) \cup \{z\}$. This means that $|O(Y)| = |O(X)| + |\mathcal{E}(X)| = 2^n$. Using the fact we know that $|\mathcal{P}(Y)| = 2^{|Y|} = 2^{n+1}$, we then know that $|\mathcal{E}(Y)| = 2^n$ since all the elements of $\mathcal{P}(Y)$ not a part of $O(Y)$ are in $\mathcal{E}(Y)$.

Therefore, we have proved that if $|O(X)| = |\mathcal{E}(X)| = 2^{|X|-1}$ for $X$, then $|O(Y)| = |\mathcal{E}(Y)| = 2^{|X|}$ where $|Y| = |X| + 1$.

Using proof by induction, we have thus shown that for $|X|$ finite and $|X| \geq 1$, $|O(X)| = |\mathcal{E}(X)| = 2^{|X|-1}$.

**2.2 8 / 8**

✓ **- 0 pts** Correct

Proof with induction

- **- 1 pts** issue with base case
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Using binomial theorem

- **- 4 pts** doesn't explain binomial theorem
- **- 2 pts** doesn't have a clear/ correct statement for the binomial theorem
- **- 2 pts** doesn't apply binomial theorem correctly/usefully



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3a) Given $X = \{0, 1, 2, 3\}$, $\mathcal{E} = \{(0,2), (1,3)\}$ $h = \{((0,2),a), ((0,2),b), ((1,3),a), ((1,3),b)\}$

3b) If $h$ is a function, this means that every $A$ only corresponds to 1 value for $y$. Thus, in order for this to possible, every $x \in A$ has to map to the same value of $y$, or else $A$ would map to multiple values, making $h$ not a function. Since $A$ is defined as an equivalent class (subset of $\mathcal{E}$), all of the values in $A$ are related by $R$. Putting this together with what we know about the outputs of values of $A$, we know that all values that can be written as an ordered pair $(x,y) \in R$ map to the same value. Therefore, if $h$ is a function, then for $x, y \in X$, if $(x,y) \in R$, then $f(x) = f(y)$.

FIVE STAR.
FIVE STAR.

3.1 **3 / 4**

$\mathcal{E}$

- **0 pts** Correct – $\{ \{0,2\}, \{1,3\}\}$ or $\{ [0],[1] \}$.

✓ - **1 pts Listed ordered pairs instead of sets**

- **1 pts** Listed with repetition

- **1 pts** Listed representatives instead of equivalence classes

- **2 pts** Incorrect

$h$

✓ - **0 pts Correct**

- **1 pts** Correct except missing $([0]_R, b)$ and/or $([1]_R, b)$

- **1 pts** Other – see comment

- **2 pts** Incorrect

Your answer for $h$ is incorrect, but would be correct if your answer for $\mathcal{E}$ were correct



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3a) Given $X = \{0, 1, 2, 3\}$, $\mathcal{E} = \{(0,2), (1,3)\}$ $h = \{((0,2), a), ((0,2), b), ((1,3), a), ((1,3), b)\}$

3b) If h is a function, this means that every A only corresponds to 1 value for y. Thus, in order for this to possible, every $x \in A$ has to map to the same value of y, or else A would map to multiple values, making h not a function. Since A is defined as an equivalent class (subset of $\mathcal{E}$), all of the values in A are related by R. Putting this together with what we know about the outputs of values of A, we know that all values that can be written as an ordered pair $(x,y) \in R$ map to the same value. Therefore, if h is a function, then for $x, y \in X$, if $(x,y) \in R$, then $f(x) = f(y)$.

3.2 **6 / 6**

Definition of $$h$$

✓ **+ 2 pts** **Correctly used**

**+ 1 pts** Uses relation from part (a) instead of a general one

**+ 1 pts** Usage not clear/not fully correct

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Overall logic

✓ **+ 2 pts** **Correct**

**+ 1 pts** Right idea but unclear/minor errors

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Definition of "function" applied to $$h$$

✓ **+ 2 pts** **Correctly used**

**+ 1 pts** Not completely correct/correctly used (e.g. didn't use uniqueness property)

**+ 0 pts** Incorrect/not used

1 What are $$A$$ and $$y$$ here?

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3 You've shown that if $$x,y\in A$$ for some $$A\in\mathcal{E}$$, then $$f(x)=f(y)$$. You still need to show (mention) that if $$(x,y)\in R$$, then $$x,y\in A$$ for some $$A\in\mathcal{E}$$.



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4a) Given $g \circ f$ is onto, we know that for $c \in Z$, there exists $a \in X$ such that $(g \circ f)(a) = g(f(a)) = c$ Thus, if $b = f(a) \in Y$, then we can simplify the previous expression to $g(b) = z$. This proves that $g$ is onto since for every element in $Z$ (the codomain) there exists a value in $Y$, the domain, that maps to the element.

4b) Let $x_1, x_2 \in X$ and assume that $f(x_1) = f(x_2)$. Using the definition of $g \circ f$, we know that $g(f(x_1)) = g(f(x_2)) = (g \circ f)(x_1) = (g \circ f)(x_2)$. However, since $g \circ f$ is one-to-one, we know that $x_1 = x_2$. Thus, $f$ must also be one-to-one since only one domain value can map to each codomain value.

**4.1 5 / 5**

- ✓ **- 0 pts** Correct
- **- 2 pts** Wrong defintion of onto
- **- 1 pts** Overall logic
- **- 2 pts** Wrong definition of composition



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4a) Given $g \circ f$ is onto, we know that for $c \in Z$, there exists $a \in X$ such that $(g \circ f)(a) = g(f(a)) = c$. Thus, if $b = f(a) \in Y$, then we can simplify the previous expression to $g(b) = z$. This proves that $g$ is onto since for every element in $Z$ (the codomain) there exists a value in $Y$, the domain, that maps to the element.

4b) Let $x_1, x_2 \in X$ and assume that $f(x_1) = f(x_2)$. Using the definition of $g \circ f$, we know that $g(f(x_1)) = g(f(x_2)) = (g \circ f)(x_1) = (g \circ f)(x_2)$. However, since $g \circ f$ is one-to-one, we know that $x_1 = x_2$. Thus, $f$ must also be one-to-one since only one domain value can map to each codomain value.

**4.2 5 / 5**

- ✓ **+ 5 pts** Correct
- **+ 2 pts** Gave/used definition of 1-1
- **+ 2 pts** Demonstrated/used definition of composition of functions



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