# 21W-MATH61-1 Exam One

# ERIC YANG

TOTAL POINTS

# 37 / 40

## QUESTION 1

# Short answer 10 pts

# 1.1 0/2

- 0 pts Correct

- **1 pts** Right idea with a computation mistake, or slightly off number

√ - 2 pts 0

# 1.2 2/2

#### ✓ - 0 pts Correct

- **1 pts** You listed the classes, not an element from each class.

- 2 pts incorrect

- 1 pts Right idea, but incorrect final answer

# 1.3 2/2

# ✓ - 0 pts Correct

- 2 pts incorrect

# 1.4 2/2

## ✓ - 0 pts Correct

- 2 pts incorrect
- 1 pts Right idea or close answer

## 1.5 2/2

## ✓ - 0 pts Correct

- 2 pts incorrect

- 1 pts Ambiguous answer.

#### **QUESTION 2**

# Even and odd sets 10 pts

# 2.1 2/2

# ✓ - 0 pts Correct

- 1 pts Missing some sets

- 2 pts blank/ several mistakes
- 1 pts should be the empty set, not the set containing the empty set

# 2.2 8/8

## ✓ - 0 pts Correct

Proof with induction

- 1 pts issue with base case
- 2 pts many issues with inductive hypothesis
- 2 pts no base case/ several issues in base case
- 1 pts issue with inductive hypothesis
- 1 pts mistake in logic of inductive step
- 2 pts many issues in logic of inductive step
- 1 pts mistake in doing inductive step
- 2 pts many issues in doing inductive step

## Counting proof

- **4 pts** fails to construct bijection between O(X) and E(X)
  - 2 pts fails to show why IO(X)I=IE(X)I
- **3 pts** many issues with bijection between O(X) and E(X)

Using binomial theorem

- 4 pts doesn't explain binomial theorem
- 2 pts doesn't have a clear/ correct statement for

the binomial theorem

- **2 pts** doesn't apply binomial theorem correctly/usefully

#### QUESTION 3

10 pts

# 3.1 **3 / 4**

\$\$\mathcal{E}\$

- **0 pts** Correct –  $\ (0,2), (1,3)) \$  or  $\ (0,[1] ) \$ 

### $\checkmark$ - 1 pts Listed ordered pairs instead of sets

- 1 pts Listed with repetition

- **1 pts** Listed representatives instead of equivalence classes

- 2 pts Incorrect

### \$\$h\$\$

### ✓ - 0 pts Correct

- **1 pts** Correct except missing \$\$([0]\_R, b)\$\$ and/or \$\$([1]\_R, b)\$\$

- 1 pts Other see comment
- 2 pts Incorrect
- Your answer for \$\$h\$\$ is incorrect, but would be correct if your answer for \$\$\mathcal{E}\$\$ were correct

## 3.2 6/6

#### Definition of \$\$h\$\$

## ✓ + 2 pts Correctly used

+ 1 pts Uses relation from part (a) instead of a

general one

- + 1 pts Usage not clear/not fully correct
- + 0 pts Incorrect/not used/didn't explain usage

## **Overall logic**

#### ✓ + 2 pts Correct

- + 1 pts Right idea but unclear/minor errors
- + 0 pts Incorrect/missing

Definition of "function" applied to \$\$h\$\$

# √ + 2 pts Correctly used

- + **1 pts** Not completely correct/correctly used (e.g. didn't use uniqueness property)
  - + 0 pts Incorrect/not used
- 1 What are \$\$A\$\$ and \$\$y\$\$ here?
- 2 element

You've shown that if \$\$x,y\in A\$\$ for some \$\$A\in\mathcal{E}\$\$, then \$\$f(x)=f(y)\$\$. You still need to show (mention) that if \$\$(x,y)\in R\$\$, then \$\$x,y\in A\$\$ for some \$\$A\in\mathcal{E}\$\$.

#### **QUESTION 4**

10 pts

# 4.1 5/5

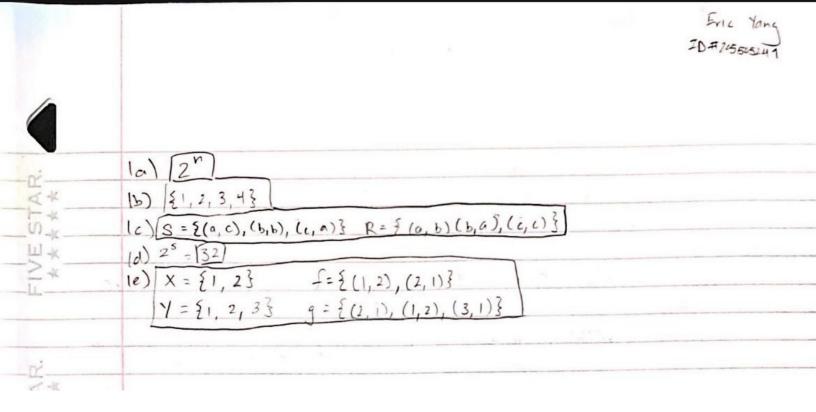
# $\checkmark$ - 0 pts Correct

- 2 pts Wrong defintion of onto
- 1 pts Overall logic
- 2 pts Wrong definition of composition

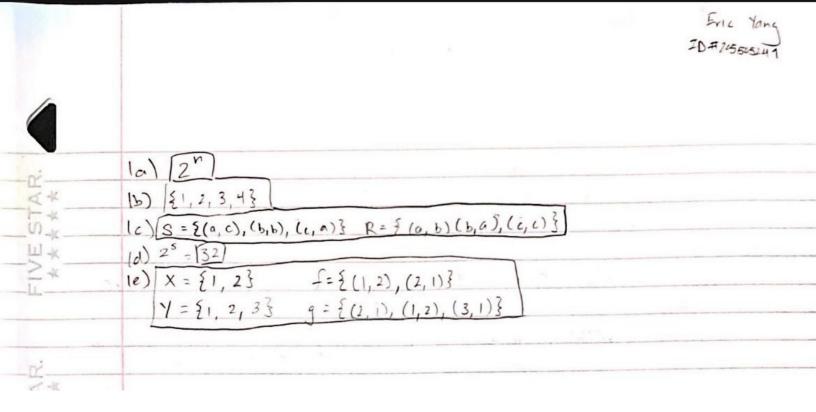
### 4.2 5/5

- ✓ + 5 pts Correct
  - + 2 pts Gave/used definition of 1-1
  - + 2 pts Demonstrated/used definition of

composition of functions

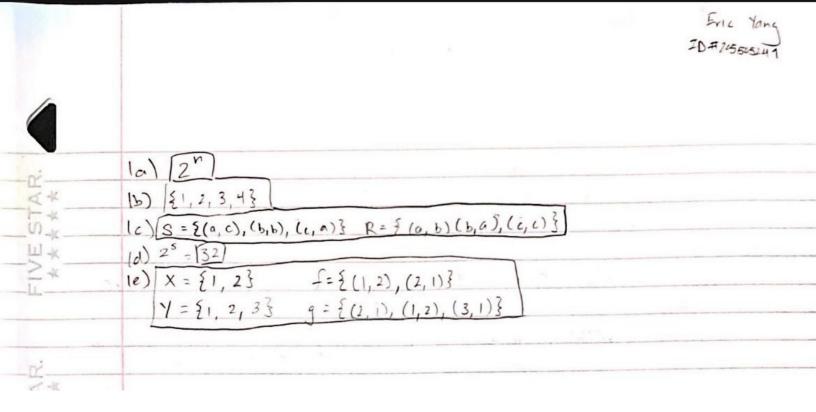


- 1.1 0/2
  - 0 pts Correct
  - 1 pts Right idea with a computation mistake, or slightly off number
  - √ 2 pts 0



# 1.2 **2 / 2**

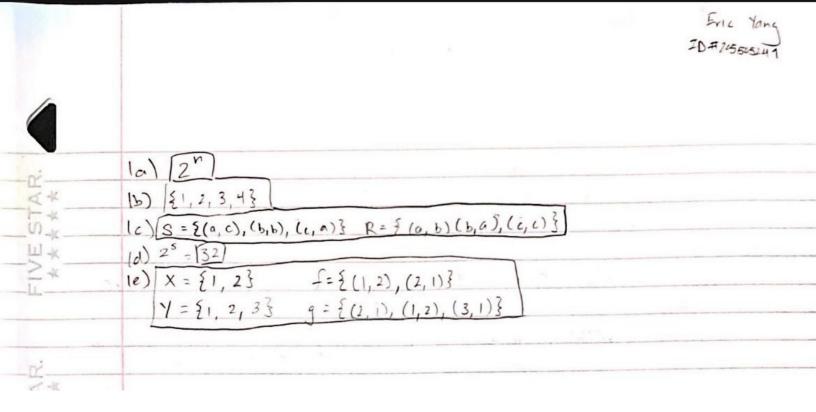
- 1 pts You listed the classes, not an element from each class.
- 2 pts incorrect
- 1 pts Right idea, but incorrect final answer



# 1.3 2/2

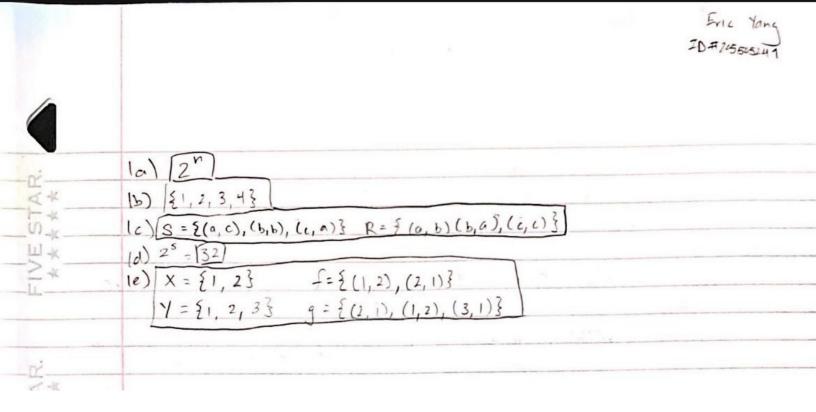
# ✓ - 0 pts Correct

- 2 pts incorrect



# 1.4 2/2

- 2 pts incorrect
- 1 pts Right idea or close answer



# 1.5 2/2

- 2 pts incorrect
- 1 pts Ambiguous answer.

=> O({a, b, c}) = {a}, {b}, {c}, {a, b, c} => E({a, b, c}) = D, {a, b}, {a, c} {b, c}

25) Base case: |x| = 1, O(x) = X,  $E(x) = |\phi| \Rightarrow |O(x)| = |E(x)| = 2^{n-1}$ . Inductive step Assume for |x| = n,  $|O(x)| = |E(x)| = 2^{n-1}$ . Let |Y| = n+1 where Y contains X and on extra element. Thus, AGP(Y)= |A| is odd, or O(Y), contains O(x) and  $E(x) \cup \{z\}$ . This means that  $|O(Y)| = |O(x)| + |E(x)| = 2^n$ . Using the fact we know that  $|P(Y)| = 2^{|Y|} = 2^{n+1}$ , we then know that  $|E(Y)| = 2^n$  since all the elements of P(Y) not a part of O(Y) are in E(Y). Therefore, we have proved that if  $|O(x)| = |E(x)| = 2^{n+1} - 1$ . then  $|O(Y)| = |E(Y)| = 2^{1\times 1}$  where |Y| = |x| + 1. Using proof by induction, we have thing shown that for |x|finite and  $|x| \ge 1$ ,  $|O(x)| = |E(x)| = 2^{N-1}$ .

# 2.1 2/2

- 1 pts Missing some sets
- 2 pts blank/ several mistakes
- 1 pts should be the empty set, not the set containing the empty set

=> O({a, b, c}) = {a}, {b}, {c}, {a, b, c} => E({a, b, c}) = D, {a, b}, {a, c} {b, c}

25) Base case: |x| = 1, O(x) = X,  $E(x) = |\phi| \Rightarrow |O(x)| = |E(x)| = 2^{n-1}$ . Inductive step Assume for |x| = n,  $|O(x)| = |E(x)| = 2^{n-1}$ . Let |Y| = n+1 where Y contains X and on extra element. Thus, AGP(Y)= |A| is odd, or O(Y), contains O(x) and  $E(x) \cup \{z\}$ . This means that  $|O(Y)| = |O(x)| + |E(x)| = 2^n$ . Using the fact we know that  $|P(Y)| = 2^{|Y|} = 2^{n+1}$ , we then know that  $|E(Y)| = 2^n$  since all the elements of P(Y) not a part of O(Y) are in E(Y). Therefore, we have proved that if  $|O(x)| = |E(x)| = 2^{n+1} - 1$ . then  $|O(Y)| = |E(Y)| = 2^{1\times 1}$  where |Y| = |x| + 1. Using proof by induction, we have thing shown that for |x|finite and  $|x| \ge 1$ ,  $|O(x)| = |E(x)| = 2^{N-1}$ .

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Using binomial theorem

- 4 pts doesn't explain binomial theorem
- 2 pts doesn't have a clear/ correct statement for the binomial theorem
- 2 pts doesn't apply binomial theorem correctly/usefully

30) Given X = 20, 1, 2, 33, E = 2(0, 2) (1, 3) 3 h= 2(10, 2), a), (10, 21, b), STAR. ((1,3), a), ((1,3), b)] SX 11-1 3b) It h is a function, this means that every A only N= corresponds to 1 value for up Thus, in order for this to possible, every x 6A has to map to the some value of y, or clse A would map to multiple values, making h not a function. STAR. \* \* \* \* Since A is defined as an equivalent class (suzet of E), all of the values in A one related by R. Putting this typether with what we know about the outputs of values of A, we know that FIVE all volves that can be written as an ordered pair (x, y) ER3 map to the same value. Therefore, if h is a function, then for x,yEX, if (x,y)ER, then f(x)=f(y).

3.1 3/4

 $\ \$ 

- **O pts** Correct –  $\ (0,2), (1,3))$  or (0,1), (3,1)

# $\checkmark$ - 1 pts Listed ordered pairs instead of sets

- 1 pts Listed with repetition
- 1 pts Listed representatives instead of equivalence classes
- 2 pts Incorrect

# \$\$h\$\$

- 1 pts Correct except missing \$\$([0]\_R, b)\$\$ and/or \$\$([1]\_R, b)\$\$
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1 What are \$\$A\$\$ and \$\$y\$\$ here?

2 element

3 You've shown that if  $$x,y\in A$  for some  $A(x,y)\in A$ , then f(x)=f(y). You still need to show (mention) that if  $f(x,y)\in A$ , then  $$x,y\in A$  for some  $A(x,y)\in A$ .

4a) Given get is onto, we know that for CGZ, there exists  $a \in X$  such that  $(g \circ f)(A) = g(f(A)) = c$  Thus, if  $b = f(A) \in Y$ , then we can simplify the previous expression to g(b) = z. This proves that g is onto since for every element in Z there exists a value in Y, the demain, that maps to the element. (the ordermain)

4b) Let x1, X2 EX and assume that f(x,)=f(x2). Using the definition of got, we know that  $g(f(x_1)) = g(f(x_2)) = (g \circ f)(x_1) = (g \circ f)(x_2)$ . However, since got is one-to-one, we know that x, = x2. Thus, f must also be one to -one since only one domain value can map to each adomain value.

4.1 5/5

- 2 pts Wrong defintion of onto
- 1 pts Overall logic
- 2 pts Wrong definition of composition

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## 4.2 5/5

# ✓ + 5 pts Correct

- + 2 pts Gave/used definition of 1-1
- + 2 pts Demonstrated/used definition of composition of functions