Midterm 1

Last Name: Miller

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12!
10·2!

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Section:

	Tuesday:	Thursday:	
	2A	2B	TA: Alex Mennen
	2C	2D	TA: Van Latimer

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators,** books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. There is some scratch paper at the back of the exam. Please circle or box your final answers. **Please get out your id and be ready to show it when you turn in your exam.**

Please do not write below this line.

Question	Points	Score
1	10	6
2	10	6
3	10	7
4	10	2
5	10	10
Total:	50	31

1. (10 points) 1. If R is a partial order on a set X, then R^{-1} is necessarily:

(a) A function

(b) Symmetric

(c) A partial order

(d) An equivalence relation

refl. antisymm

trans

2. If X has 5 elements and Y has 3 elements, the number of relations from X to Y is:

(a) $2^5 2^3$

(b) $2^5 + 2^3$

(c) $2^{5\times 3}$

(d) $2^5 3$

3. $\frac{10!}{8!}$ is

(a) more that 10 but less than 50

(b) more than 50 but less than 100

(c) more than 100 but less than 200

(d) more than 200

10 · 9

4. The function $f : \{a, b\}^* \to \{a, b\}^*$ (X^* is the set of strings in X) defined by $f(\alpha) = \alpha\alpha$ is:

(a) onto but not one-to-one

(b) one-to-one but not onto

(c) onto and one-to-one

(d) neither one-to-one nor onto

one to one

5. If $X = \{1, 2, 3, 4\}$ then the relation $R = \{(2, 3), (3, 2), (2, 2)\}$ is:

(a) ~~Reflexive~~

(b) Transitive

(c) Symmetric

(d) Antisymmetric

2. In this question write down your answer, no need for any justification. Please leave answers in the form of factorials, $C(n,m)$ or $\binom{n}{m}$, $P(n,m)$, and with exponents instead of multiplying the answer out.

(a) (2 points) What is the coefficient of x^2y^6 in $(2x-3y)^8$?

$$\boxed{\binom{8}{6}(2)^2(3)^6}$$

8 chose 6

✓

(b) (2 points) What is the number of ways to arrange seven math majors and five CS majors in a line, if the CS majors will not stand next to each other?

7 math 5 CS

$5!+7!+4!$

5 7 4 6 3 5 2 4 1

✗

(c) (2 points) What is the number of ways of rearranging the letters of COMBINATORICS?

C 2
O 2
I 2

$$\boxed{\frac{13!}{2!2!2!}}$$

✓

(d) (2 points) What is the number of one-to-one functions from $\{1,2,3\}$ to $\{a,b,c,d,e\}$ such that a and b are both in the range of the function?

(e) (2 points) What is the number of way to distribute 11 identical acorns between 3 different squirrels?

d. 1 2 3 a b c d e

3 + ✗ a b 3

e. $\frac{(n+r-1)!}{n!(r-1)!}$

$$\boxed{\frac{13!}{11!\,2!}}$$

✓

3. Consider the relation on the real numbers defined by xRy if $x - y$ is an integer.

(a) (6 points) Show that R is an equivalence relation.

equivalence = ~~tran~~ reflexive, transitive, symmetric

refl. every x ~~can be minused by $(x-1)$~~ ?

~~$x - (x-1) = 1$ = an integer~~ -2

trans. if $x - y$ int then $y - z$ int

if $x-y$ makes integer, then it is 1 or more less or greater if y makes int then 1 or more greater, same decimals for this to be true $\boxed{\text{TRANS}}$ eg. $1.25 - 0.25$ same decimals

$0.25 - 2.25$

$x - y$

negative numbers allowed

$x - (x-1)$ = integer $(x-1) - (x)$ = negative integer

∴ symmetric

-1

(b) (4 points) Write down 3 distinct elements in the equivalence class of 1/4.

$[\frac{1}{4}] = \{1.25, 2.25, 3.25\}$

4. (a) (5 points) Show that if $f : X \to Y$ and $g : Y \to Z$ are functions and f and g are both onto, then $g \circ f$ is onto. Justify your work!

$f(X) = Y \quad G(Y) = Z$

f onto $\quad$ g onto

?

1/5 $\quad$ ~~defd~~ means there is an inverse funct.

all range used

if $g(f(X))$

$g(Y) = Z$

? $g(f(Y)) = Z$ — goes to all range of g same as $g(Y)=Z$ so hence it is also onto

(b) (5 points) Show that $\sum_{i=0}^{n} 2^i \binom{n}{i} = 3^n$ (remember $\binom{n}{i}$ means the same thing as $C(n, i)$). Justify your work!

1/5

$\sum_{i=0} 2^i \binom{n}{i} = 3^n$

$3^n = (1+2)^n =$

$2^0 \binom{0}{0} = 3^0$ base case

$1 = 1$

? $\quad 2^i \binom{n}{i} + 2^i \binom{n+1}{i} = 3^{n+1}$ — inductive step

true ∴ by induction ?

$2^i \frac{n!}{i!(n-i)!} + 2^i \frac{(n+1)!}{i!(n+1-i)!} = 3 \cdot 3^n = 2 \cdot 3^n + 1 \cdot 3^n$

5. (10 points) Use induction to show that $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$. Be sure to show your work and justify your answer.

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

base case

$i=1$

$$1 = \left(\frac{1(1+1)}{2}\right)^2$$

$$1 = 1$$

✓ base case works ✓

— inductive step

$$1, 8, 27 \ldots n^3 \stackrel{?}{=} \left(\frac{n(n+1)}{2}\right)^2$$

assume for n and try $n+1$

$3 \cdot 2 \cdot 4 \cdot 1$

$2+4$

$2(\cancel{x}+2)$

$$\underbrace{1, 8, 27 \ldots n^3}, + (n+1)^3 \stackrel{?}{=} \left(\frac{(n+1)(n+2)}{2}\right)^2$$

$$\left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$$

$$\stackrel{?}{=}$$

$$(n+1)^2\left(\left(\frac{n}{2}\right)^2 + \frac{4}{4}(n+1)\right)$$

$n^2+4n+4 =$

$$(\cancel{2}n+1)^2\left(\frac{(n+2)^2}{4}\right) = \left(\frac{(n+1)(n+2)}{2}\right)^2 = \left(\frac{(n+1)(n+2)}{2}\right)^2$$

✓ ✓ true for $n+1$ this is TRUE