MIDTERM 2 (MATH 61, SPRING 2017)

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Math 61 Section: 1C

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The rules:
You MUST simplify completely and BOX all answers with an **INK PEN**.
You are allowed to use only this paper and pen/pencil. No calculators.
No books, no notebooks, no phones, no web access. You MUST write your name and UCLA id. Except for the last problem, you MUST write out your logical reasoning and/or proof in full. You have exactly 50 minutes.

Warning: those caught violating the rules get automatic 10% score deduction.

Points:

1	15
2	20
3	16
4	5
5	30

Total: 86 **(out of 100)**

Problem 1. (15 points)

Solve the following LHRR: $a_{n+1} = 5a_n - 6a_{n-1}$, $a_1 = 9$, $a_2 = 20$.

$\lambda^2 = 5\lambda - 6$

$\lambda^2 - 5\lambda + 6 = 0$

$(\lambda - 2)(\lambda - 3)$

$\lambda = \{2, 3\}$

$a_n = \alpha(2)^n + \beta(3)^n$

$a_1 = \alpha(2) + \beta(3) = 9$

$a_2 = \alpha(4) + \beta(9) = 20$

$4\alpha + 6\beta = 18$

$-\ 4\alpha + 9\beta = 20$

$-3\beta = -2$

$\beta = 2/3$

$2\alpha + \cancel{3}(\frac{2}{\cancel{3}}) = 9$

$2\alpha = 7$

$\alpha = \frac{7}{2}$

$$\boxed{a_n = \frac{7}{2}(2)^n + \frac{2}{3}(3)^n}$$

$14 + 6 = 20$ ✓

$7 + 2 = 9$

✓ 15

Problem 2. (20 points)

Find the number of subgraphs of G isomorphic to H, where

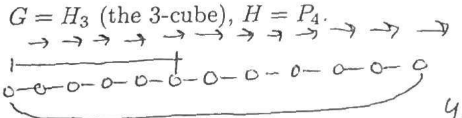
a) $G = C_{12}$, $H = P_5$.

b) $G = K_{9,9}$, $H = C_6$.

c) $G = K_{9,9}$, $H = C_7$.

d) $G = H_3$ (the 3-cube), $H = C_4$.

e) $G = H_3$ (the 3-cube), $H = P_4$.



✓ 4 a) $\boxed{12}$

You can start P_5 at any of the 12 vertices of C_{12} for a unique P_5

✓ 4 b) $\boxed{\cancel{\binom{9}{3}\binom{9}{3}}}$

~~You can pick any 3 vertices from each side and have a cycle.~~

~~You select 3 vert~~

$\# C_6 = \frac{1}{6}(\# P_6)$ since we can remove 6 edges from C_6 to get P_6

for P_6 you ~~select~~ have 9 choices then 9 then 8 then 8 then 7 then 7

so its $\boxed{\frac{1}{6}(9 \cdot 9 \cdot 8 \cdot 8 \cdot 7 \cdot 7)}$

also $\binom{9}{3}\binom{9}{3}$ since you can pick any 3 vert from each side and have cycle. same thing.

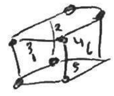
✓ 4 c)

path doesn't exist.

since it's a bipartite graph you can't complete a cycle on 7 vertices

$\boxed{0}$

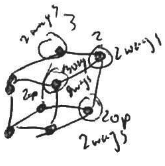
✓ 4 d)



$\boxed{6}$

because of how the 3-cube is set up there can only be 6 C_4's ~~which are~~ as imaged to the left, it is at each face since there are no diagonals there are no other possibilities.

✓ 4 e)



8 ways to choose a vertex then 3 ways for next since it's connected to 3 then 2 ways then 2 ways then divide by two for double counting start and finish condition

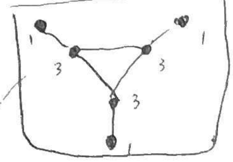
$$\boxed{\frac{8 \cdot 3 \cdot 2 \cdot 2}{2}}$$

Problem 3. (20 points)

For each of these sequences, either draw a simple graph with this score (degree sequence), or explain why there is no such graph.

a) (3,3,3,1,1,1)
b) (4,4,4,1,1,1)
c) (5,5,5,3,2,1)
d) (4,4,. . .,4) ← 16 numbers.
e) (3,3,3,3,2,2,2,2)

a)

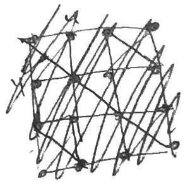


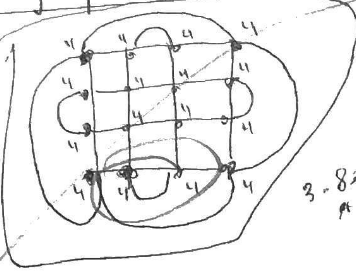
b) the ~~#of~~ sum of the degrees is odd (15) so by handshake thrm. such a graph cannot exist.

c) Σ# degree is 21 which is odd so by handshake thrm. such a graph cannot exist

→ also you can't have 3 ~~graphs~~ vert. connected to all and 1 connected to 1, it's a contradiction

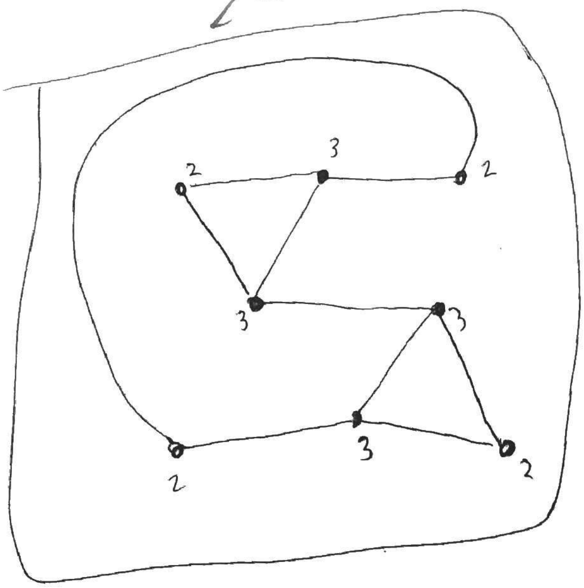
d)





3 · 8 ~~18~~ ≠ 26

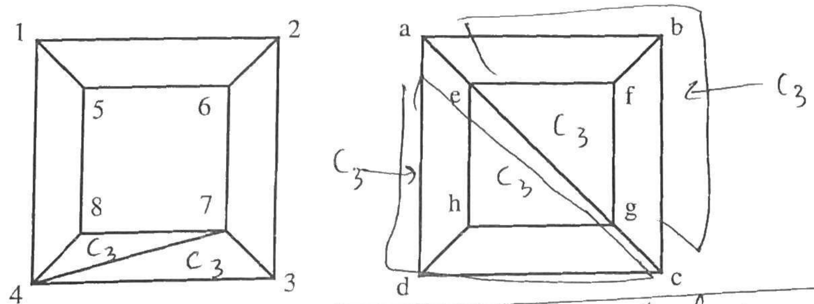
e)



Problem 4. (15 points)

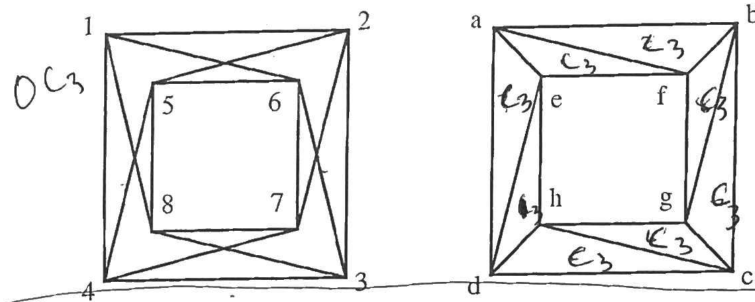
Decide whether the following pairs of graphs on 8 vertices are isomorphic or non-isomorphic.

a)



non-isomorphic, isomorphic graphs have same # of isomorphic subgraphs, but the left one has 2 C_3's while the right has 4. ✓

b)



non-isomorphic, isomorphic graphs have same # of isomorphic subgraphs, but left has 0 C_3 and right has 8 C_3's

Important: In case of isomorphism, you must present a bijection. In case non-isomorphism, you must present an argument.

Problem 5. (30 points, 2 points each) **TRUE or FALSE?**

Circle correct answers with ink. No explanation is required or will be considered.

T F (1) The hypercube graph H_4 contains an Eulerian cycle.

T F (2) The hypercube graph H_4 contains a Hamiltonian cycle.

T F (3) A subgraph of a connected graph is always connected. C,

T F (4) A subgraph of a disconnected graph is always disconnected.

T F (5) The sum of degrees of $K_{\ell,\ell}$ is ℓ^2. 5 5

T F (6) The sum of degrees of a graph on $n \geq 3$ vertices is smaller than $n^2 - 1$.

T F (7) Computing the number of walks of given length in a graph can be done efficiently.

T F (8) Deciding whether a graph has Hamiltonian cycle can be done efficiently.

T F (9) The number of (shortest) grid walks $(0,0) \to (5,5)$ which do not go through $(1,3)$, $(3,4)$, $(4,1)$, $(1,5)$ is > 100.

T F (10) Graph H is a subgraph of G. Graph H contains a Hamiltonian cycle. Then G contains a Hamiltonian cycle.

T F (11) Graph H is a subgraph of G. Graph G contains a Hamiltonian cycle. Then H contains a Hamiltonian cycle.

T F (12) Graph $K_{50,50}$ contains a subgraph isomorphic to K_{10}.

T F (13) Graph $K_{50,50}$ contains a subgraph isomorphic to $K_{10,10}$.

T F (14) The number of walks $1 \to 1$ of length k in a graph G on n vertices can be computed via matrix $(A_G)^n$.

T F (15) Isomorphic graphs have the same number of Eulerian circuits.

K_{10} $10 \cdot 9$