MIDTERM 2 (MATH 61, SPRING 2017)

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Math 61 Section: 1B

Date: 5/19/17

The rules:
You MUST simplify completely and BOX all answers with an **INK PEN**. You are allowed to use only this paper and pen/pencil. No calculators. No books, no notebooks, no phones, no web access. You MUST write your name and UCLA id. Except for the last problem, you MUST write out your logical reasoning and/or proof in full. You have exactly 50 minutes.

Warning: those caught violating the rules get automatic 10% score deduction.

Points:

1	15
2	18
3	20
4	15
5	28

Total: 96 **(out of 100)**

Problem 1. (15 points)

Solve the following LHRR: $a_{n+1} = 5a_n - 6a_{n-1}$, $a_1 = 9$, $a_2 = 20$.

$\lambda^2 = 5\lambda - 6$

$0 = \lambda^2 - 5\lambda + 6$

$= (\lambda - 2)(\lambda - 3)$

$\lambda_1 = 2 \quad \lambda_2 = 3$

$a_n = \alpha_1 (\lambda_1)^n + \alpha_2 (\lambda_2)^n$

$a_1 = 9 = \alpha_1 (2) + \alpha_2 (3)$

$a_2 = 20 = \alpha_1 (4) + \alpha_2 (9) \quad = \quad 14 + 6$

$2 = 3\alpha_2$

$\alpha_2 = \frac{2}{3}$

$9 = 2\alpha_1 + 2$

$\alpha_1 = \frac{7}{2}$

$a_n = \frac{7}{2}(2)^n + \frac{2}{3}(3)^n$

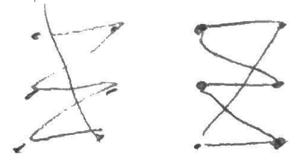
$\boxed{a_n = \frac{7}{2}(2)^n + \frac{2}{3}(3)^n}$

Problem 2. (20 points)

Find the number of subgraphs of G isomorphic to H, where

a) $G = C_{12}$, $H = P_5$.
b) $G = K_{9,9}$, $H = C_6$.
c) $G = K_{9,9}$, $H = C_7$.
d) $G = H_3$ (the 3-cube), $H = C_4$.
e) $G = H_3$ (the 3-cube), $H = P_4$.

$$\frac{\binom{8}{4}\cdot 3\cdot 2\cdot 2}{2}$$



$3\cdot 2\cdot 2\cdot$ 12

a. each vertex we choose as a start for the path
4 determines every ~~other~~ remaining vertex going around the cycle in one direction, which we do to avoid double counting.
Thus, $\boxed{12}$

b. we choose 3 vertices from each side. we then ~~have~~
2 choose a first vertex and have 6 ways of doing so. We have a total of $\frac{6\cdot 3\cdot 2\cdot 2}{6}$ ~~vert paths~~ cycles to make from our 6 vertices

$(\cancel{3\cdot 2\cdot 2})\cancel{\binom{9}{3}}\cancel{\binom{9}{}}$ $\boxed{12\binom{9}{3}\binom{9}{3}}$

c. No cycles exist. we would be stranded on the wrong
4 side of the bipartite graph $\boxed{0}$

d. Each face of the cube is a C_4.
4 $\boxed{6}$

e. ~~the~~ We choose our first vertex of 8, and we have 3
4 possible paths, followed by 2, followed by 2. divide by two for double counting.

$$\frac{8\cdot 3\cdot 2\cdot 2}{2} = \boxed{8\cdot 3\cdot 2}$$

Problem 3. (20 points)

For each of these sequences, either draw a simple graph with this score (degree sequence), or explain why there is no such graph.

$4 \cdot 16 = 64$

$E = 32$

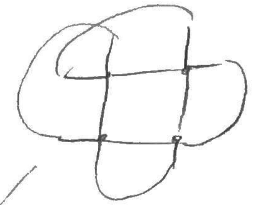
a) (3,3,3,1,1,1)

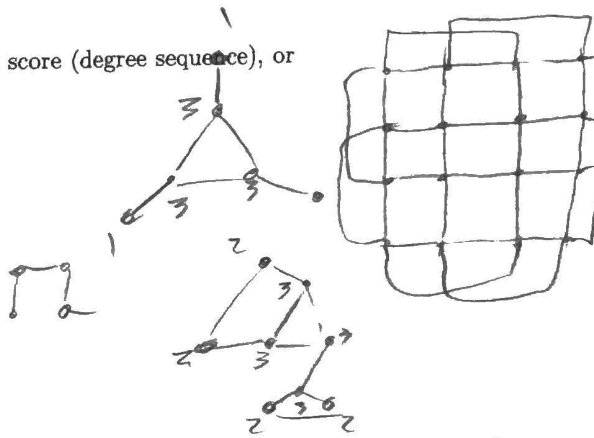
b) (4,4,4,1,1,1)

c) (5,5,5,3,2,1)

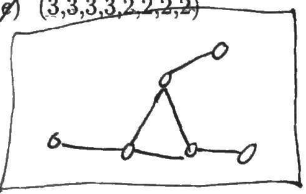
d) (4,4,...,4) ← 16 numbers.

e) (3,3,3,3,2,2,2,2)





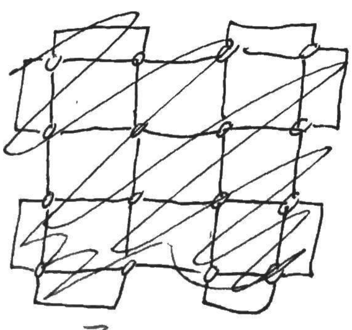
a.

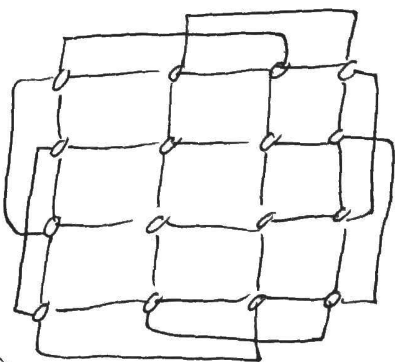


b. This graph cannot exist by the Handshake Thm, which states the sum of the degree sequence must be ~~odd~~ even.

c. This graph cannot exist. Since three vertices have scores of one less than the total number of vertices, they each must connect to all of the other vertices in the graph. Thus, we cannot have a vertex of degree 1.

d.





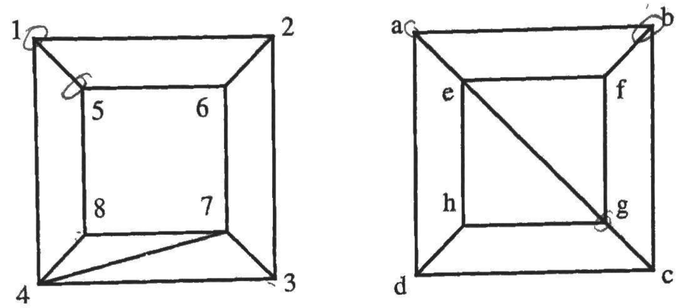
e.

Problem 4. (15 points)

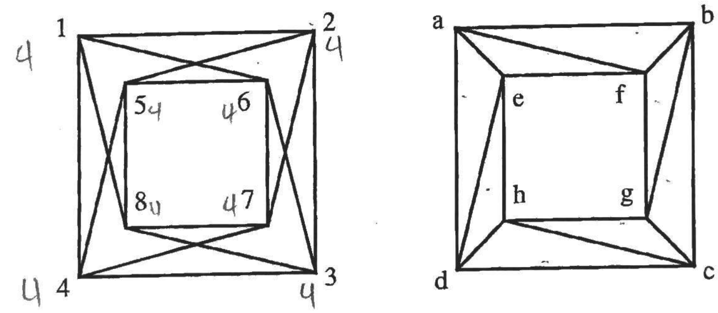
Decide whether the following pairs of graphs on 8 vertices are isomorphic or non-isomorphic.

1 → a
2 → d
3 → h
4 → e
5 → b
6 → c
7 → g
8 → f

a)



b)



Important: In case of isomorphism, you must present a bijection. In case non-isomorphism, you must present an argument.

a. bijection: 1 → a
2 → d
3 → h
4 → e
5 → b
6 → c
7 → g
8 → f

isomorphic ✓

b. ~~This is not~~ These graphs are not isomorphic because the graph on the left contains ~~a tr~~ no 3-cycles, while the graph on the right has 8 C_3's. ✓

Problem 5. (30 points, 2 points each) **TRUE or FALSE?**

Circle correct answers with ink. No explanation is required or will be considered.

T F (1) The hypercube graph H_4 contains an Eulerian cycle.

T F (2) The hypercube graph H_4 contains a Hamiltonian cycle.

T F (3) A subgraph of a connected graph is always connected.

T F (4) A subgraph of a disconnected graph is always disconnected.

T F (5) The sum of degrees of $K_{\ell,\ell}$ is ℓ^2. $\ell(\ell)+\ell(\ell)$

T F (6) The sum of degrees of a graph on $n \geq 3$ vertices is smaller than $n^2 - 1$.

T F (7) Computing the number of walks of given length in a graph can be done efficiently.

T F (8) Deciding whether a graph has Hamiltonian cycle can be done efficiently.

T F (9) The number of (shortest) grid walks $(0,0) \to (5,5)$ which do not go through $(1,3)$, $(3,4)$, $(4,1)$, $(1,5)$ is > 100. $\binom{10}{5}$

63 · 4

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T F (10) Graph H is a subgraph of G. Graph H contains a Hamiltonian cycle. Then G contains a Hamiltonian cycle.

T F (11) Graph H is a subgraph of G. Graph G contains a Hamiltonian cycle. Then H contains a Hamiltonian cycle. $\binom{5}{1}$ 70

T F (12) Graph $K_{50,50}$ contains a subgraph isomorphic to K_{10}. 5

T F (13) Graph $K_{50,50}$ contains a subgraph isomorphic to $K_{10,10}$.

T F (14) The number of walks $1 \to 1$ of length k in a graph G on n vertices can be computed via matrix $(A_G)^n$.

T F (15) Isomorphic graphs have the same number of Eulerian circuits.