Problem 1. (20 points)

Compute the probability that 4-subset A of $\{1,2,\ldots,10\}$ satisfies:

a) A has no odd numbers,

b) A has at least one number ≤ 3 ,

c) A contains 1 but not 7.

d) the smallest number in A is divisible by 3

Problem 2. (20 points)

Let $X = \mathbb{N} = \{0, 1, 2, \ldots\}$ be the set of all non-negative integers. For each of the following functions $f: X \to X$ decide whether they are mjective, surjective, bijective:

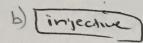
$$a) \quad f(x) = x + 1$$

$$b) \quad f(x) = x^2 - 1$$

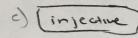
$$c) \quad f(x) = 2x$$

d)
$$f(x) = (x^2 + 2x)/(x+2)$$

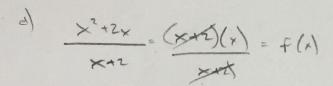




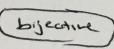












4

-2

1111 = 101

MIDTERM 1 (MATH 61, SPRING 2017)

Problem 3. (15 points)

4

Let $a_n = 1111 \cdots 1$ (n ones). Suppose a_k is divisible by 97. Use induction to show that $a_{k\cdot n} = 0 \mod 97$, for all $n \ge 1$.

D: Induction

Base: 2 = 0 mod 97

Inductive Step: 2k.n = de (n+1) = 0 mod 97

3k.n=0 mod 97 = 3k(n+1) = 0 mod 97

MIDTERM 1 (MATH 61, SPRING 2017)

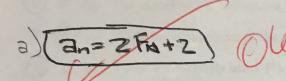
Problem 4. (15 points)

Find closed formulas for the following sequences:

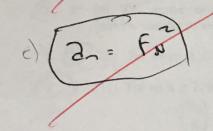
- a) $4, 4, 6, 8, 12, 18, 28, 42, 70, 112, \dots$
- b) $a_1 = 1$, $a_{n+1} = a_n \cdot \binom{n+1}{2}$
- c) $a_1 = 1$, $a_2 = 1$, $a_{n+1} = a_{n-1} a_n$ for $n \ge 2$.

c) $a_1 = 1$, $a_2 = 1$, $a_{n+1} - a_{n-1}$ $a_n = 1$ Note: you can express a_n in terms of Fibonacci numbers F_n .

2,=1 } 22=1 (2)=3 | 23=3 (2)=18







10

Problem 5. (30 points, 2 points each) TRUE or FALSE?

Circle correct answers with ink. No explanation is required or will be considered.

T (1) The number of functions from $\{A, B, C, D\}$ to $\{1, 2, 3\}$ is equal to 4^3 .

T $\stackrel{\frown}{\mathbf{F}}$ (2) The sequence $1, 3/2, 5/3, 7/6, 9/8, \dots$ is increasing.

 \mathbf{T} \mathbf{F} (3) The sequence $-1, -2, -3, -4, \dots$ is non-increasing.

T $\stackrel{\frown}{\mathbf{F}}$ (4) There are 4 anagrams of the word MAMA.

F (5) There are infinitely many Fibonacci numbers which are divisible by 3.

F (6) The number of permutations of $\{1,2,3,4,5\}$ is smaller than 123.

T F (7) The number of 3-permutations of $\{1, 2, 3, 4, 5, 6\}$ is equal to $\binom{6}{3}$.

F (8) The number of 3-subsets of $\{1,2,3,4\}$ is equal to 4.

T F (9) The number of permutations of $\{1, 2, ..., n\}$ which have n preceding n-1 (not necessarily immediately) is equal to n!/2

T For every $A, B \subset \{1, 2, ..., 12\}$ we have $|A \cap B| < |A \cup B|$.

T F (11) For all $n \ge 1$, we have

$$\binom{2n}{0} + \binom{2n}{2} + \binom{2n}{4} + \ldots + \binom{2n}{2n} = 2^{2n-1}.$$

T F (12) The number of grid walks from (0,0) to (10,10) going through (3,7) is equal to $\binom{10}{3}^2$.

T (13) The number of grid walks from (0,0) to (10,10) avoiding (10,0) and (0,10) is $\left(\frac{20}{10}\right)$ equal to $\frac{1}{2}\binom{20}{10}$.

T (14) The number of anagrams of MISSISSIPPI which begin with M is greater than the number of anagrams which begin with S.

F (15) The following parabolas are drawn in the plane:

$$y = x^2 - n^2x - n^3$$
, $n = 1, ..., 12$.

Then the regions of the plane separated by these parabolas can be colored with two colors in such a way that no two adjacent regions have the same color.