Math 61 MT1

JUSTIN CHAO

TOTAL POINTS

36 / 53

QUESTION 1

15 pts

1.1 5 / 5

✓ - 0 pts Correct

1.2 5 / 5

✓ - 0 pts Correct

1.3 5 / 5

✓ - 0 pts Correct

QUESTION 2

10 pts

2.1 2.5 / 5

✓ - 1.5 pts No quantification

✓ - 1 pts Slightly wrong statement

2.2 4.5 / 5

✓ - 0 pts Correct

- 0.5 Point adjustment

P(n+1) is a proposition, not a quantity!

QUESTION 3

10 pts

3.1 2 / 5

✓ - 1 pts omission/error in the scope of variables (in the CONIDITION part of existence): [number], [divisor] are integers, [divisor] is nonzero.

✓ - 1 pts omission/error in the scope of variables (in the CONSEQUENCE part of existence): [quotient], [remainder] are integers, 0 <= [remainder] < absolute value of [divisor]

✓ - 1 pts omission/error in the scope of variables (in the CONDITION part of uniqueness): [quotient1], [quotient2] [remainder1], [remainder2] are integers, 0 <= [remainder1], [remainder2] < absolute value of [divisor]

3.2 0 / 5

✓ - 5 pts Incorrect (not on the right track)

QUESTION 4

18 pts

4.1 2 / 2

✓ - 0 pts Correct

4.2 2 / 2

✓ - 0 pts Correct

4.3 2 / 2

✓ - 0 pts Correct

4.4 2 / 2

✓ - 0 pts Correct

4.5 1 / 2

✓ - 1 pts Blank

4.6 2 / 2

✓ - 0 pts Correct

4.7 0 / 2

✓ - 2 pts Incorrect

4.8 0 / 2

✓ - 2 pts Incorrect

4.9 1 / 2

✓ - 1 pts Blank

Math 61, Winter 2020
Introduction to Discrete Structures
Midterm Exam 1

January 29, 2020

Please show your work. You will receive little or no credit for a correct answer to a problem which is not accompanied by sufficient work/explanations, unless the problem explicitly states that no work is necessary. You can not use any notes, books, or electronic devices of any kind during the exam. If you have a question about any particular problem, please raise your hand. At the completion of the exam, please hand the exam booklet. If you have any questions about the grading of the exam, please consult the midterm regrade policy in the course syllabus.

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I certify that the work appearing on this exam is completely my own.

Signature: Justin Chao

Question:	1	2	3	4	Total
Points:	15	10	10	18	53
Score:					

$P \to Q$ $\neg P \vee Q$ $\neg P \wedge Q$

P	Q	$\neg P$	$P \to Q$	$\neg P \vee Q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

1. Do each of the following and provide justification where applicable:
   (a) Define what it means for a binary relation $R \subseteq X \times X$ on a set X to be a *total order*. (5)
   (b) Give the negation of the following proposition in an equivalent form with **no negation symbols:** (5)

$$\forall a\ \forall b\ \forall c\ \exists x\ \left(a \neq 0 \rightarrow\ ax^2 + bx + c = 0\right)$$

   The domain of discourse for every variable is $\mathbb{R}$.
   (c) Determine whether the following proposition is a tautology, a contradiction, or neither: (5)

$$(p \rightarrow r) \rightarrow ((p \rightarrow q) \wedge (q \rightarrow r))$$

(a) It means R is reflexive $(\forall x \in X,\ xRx)$
antisymmetric $(\forall x \in X\ \forall y \in X,\ xRy \wedge yRx \rightarrow x = y.)$
transitive $(\forall x \in X\ \forall y \in X\ \forall z \in X,\ xRy \wedge yRz \rightarrow xRz)$
Satisfies totality $(\forall x \in X\ \forall y \in X,\ xRy \vee yRx)$

(b) $\neg(\forall a \forall b \forall c \exists x (a \neq 0 \rightarrow ax^2 + bx + c = 0)$

$\exists a \exists b \exists c \forall x \neg(\neg a \neq 0 \vee ax^2 + bx + c = 0)$

$\exists a \exists b \exists c \forall x (\neg\neg a \neq 0 \wedge \neg ax^2 + bx + c = 0)$

$\boxed{\exists a \exists b \exists c \forall x (a \neq 0 \wedge ax^2 + bx + c \neq 0)}$
The domain of discourse forevery variable is $\mathbb{R}$

(c)

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$(p \to r) \to ((p \to q) \wedge (q \to r))$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	F
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\boxed{\text{neither}}$

2. Do the following:

   (a) State the *Binomial Theorem.* (5)

   (b) Prove for every $n \geq 1$ that (5)

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}.$$

(a): Binomial Theorem is the following:

$$(a \boxed{-} b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k.$$

(b) Let $P(n)$: "$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$ for $n \geq 1$"

Base Case: We want to prove $P(1)$ holds.

$$\sum_{k=1}^{1} \frac{1}{k(k+1)} = \frac{1}{1(1+1)} = \frac{1}{2} \qquad \frac{1}{1+1} = \frac{1}{2}$$

Since both the left hand size & right hand side equals ½, $P(1)$ holds.

Inductive step: Assume $P(n)$ holds. We want to prove $P(n+1)$ holds.

$$\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \frac{1}{(n+1)(n+2)} + \underbrace{\sum_{k=1}^{n} \frac{1}{k(k+1)}}_{P(n)}$$

$$= \frac{1}{(n+1)(n+2)} + \frac{n}{n+1}$$

$$= \frac{1}{(n+1)(n+2)} + \frac{n(n+2)}{(n+1)(n+2)}$$

$$= \frac{n(n+2)+1}{(n+1)(n+2)}$$

$$= \frac{n^2+2n+1}{(n+1)(n+2)}$$

$$= \frac{(n+1)^{\cancel{2}}}{\cancel{(n+1)}(n+2)}$$

$$= \frac{n+1}{n+2} = \frac{n+1}{(n+1)+1}$$

Since we expected $\boxed{P(n+1) = \frac{n+1}{(n+1)+1}}$ and we proved it is true, $P(n)$ holds by induction.

[redacted]

3. Do the following:
   (a) State the *Quotient-Remainder Theorem.* (5)
   (b) Prove that every number of the form $4k^2 - 5$ (where $k \in \mathbb{Z}$) cannot be written as the sum of two squares of integers. In other words, prove that for every $x, y, k \in \mathbb{Z}$, we have $x^2 + y^2 \neq 4k^2 - 5$. (5)

(a) The Quotient-Remainder Theorem states that for all $d, n \in \mathbb{Z}$ $\exists q, r \in \mathbb{Z}$ such that $n = dq + r$ (existence) and if $q_0, q_1, r_0, r_1 \in \mathbb{Z}$ and $n = dq_0 + r_0 = dq_1 + r_1$ then $q_0 = q_1$ and $r_0 = r_1$. (uniqueness).

(b)

If we set $x^2 + y^2 = n$, then by the quotient remainde theorem, $n = 4q + r$, where $q, r \in \mathbb{Z}$. Furthermore, $0 \leq r < 4$. Thus, we can have the following cases:

$n = 4q$

$n = 4q + 1$

$n = 4q + 2$

$n = 4q + 3$.

There is no integer q that can satisfy any of these cases if $n = 4k^2 - 5$ as well, because we would need to manipulate k to not be an intege to satisfy the cases.

$4q = 4k^2 - 5$

$\frac{4k^2}{4} = \frac{4q + 5}{4}$

$k^2 = q + 5/4$

$k = \pm\sqrt{q + 5/4}$

$4q + 1 = 4k^2 - 5$

$\frac{4k^2}{4} = \frac{4q + 6}{4}$

$k^2 = q + 3/2$

$k = \pm\sqrt{q + 3/2}$

2

4. For each of the following statements, indicate whether they are True or False. **A blank answer will receive 1 point.** [Recall: True means the same thing as "always true" and False means the same thing as "there exists a counterexample".] No work is necessary for this problem.

(a) False For all sets X, Y, we have $\overline{X \cup Y} = \overline{X} \cup \overline{Y}$. (2)

(b) True Suppose A, B, C, D are sets such that $A \subseteq B \subseteq C \subseteq D \subseteq A$. Then $A = C$. (2)

$\{\emptyset\}$

(c) False The cardinality of $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$ is 3. (2)

(d) True Suppose $R \subseteq \mathbb{N} \times \mathbb{N}$ is a binary relation on $\mathbb{N}$. Then the proposition:

$$(\forall x\ \forall y\ R(x,y)) \to (\forall x\ \exists y\ R(x,y))$$

(where the domain of discourse is $\mathbb{N}$) is true. (2)

(e) ______ For every $a \in \mathbb{Z}$, $6|a(a+1)(a+2)$. (2)

(f) False Every binary relation R on $\mathbb{N}$ is either symmetric or antisymmetric. (2)

(g) True For every $a, b, c \in \mathbb{Z}$ and $n \geq 1$, if $ab \equiv ac \pmod{n}$, then $b \equiv c \pmod{n}$. (2)

$n | a(b-c)$ $n | b-c$

(h) False The relation $\{(5,2),(1,6)\}$ is transitive. (2)

(i) ______ Suppose $P(n)$ is a property that a natural number $n \geq 1$ may or may not have. Furthermore, suppose that

1. $P(1)$ is true. 1, 2,
2. For every $n \geq 1$ and every prime number p, the implication $P(n) \to P(pn)$ is true.

Then $P(n)$ is true for every $n \geq 1$. (2)