Math 61
Fall 2017
11/20/17
Time Limit: 50 Minutes

Name (Print)
SID Number

Day \ T.A.	Eric	David	Ch
Tuesday	2A	2D	2[illegible]
Thursday	2B	2C	2E

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any page are missing. Enter your name and SID number on the top of this page, circle your section, and pu your initials on the top of every page, in case the pages become separated.

Calculators or computers of any kind are not allowed. You are not allowed to consult any othe materials of any kind, including books, notes and your neighbors. You may use the back of th sheet for your notes ("scratch paper"). If you need additional paper, let the proctors know.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a result from class, discussion, or homework you must indicate this and explain** why the result may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	2	2
2	2	1
3	3	2
4	3	1
5	3	0
6	2	2
Total:	15	8

Of course, if you have a question about a particular problem, please raise your hand and one of the proctors will come and talk to you.

1. (2 points) The ~~summand~~ sum of coefficients not involving x in the expansion of

$$\left(\overset{a}{\frac{x}{2}}+\overset{b}{\frac{k}{x}}\right)^6$$

is 20. What is k?

$$(a+b)^6 = \sum_{i=0}^{k} \binom{6}{k} a^k b^{n-k}$$

$$\cancel{\sum_{i=0}^{k}\binom{6}{k} = 20}$$

$$\frac{6!}{5!\,1} \qquad \frac{6!}{4!\,2!} \qquad \frac{6!}{3!\,3!} \qquad \frac{6\times5\times4}{6}$$

$$\binom{6}{0}a^0b^6+\binom{6}{1}a^1b^5+\binom{6}{2}a^2b^4+\binom{6}{3}a^3b^3$$

$$+\binom{6}{4}a^4b^2+\binom{6}{5}a^5b+\binom{6}{6}a^6$$

$$\left(\frac{k}{x}\right)^6+6\left(\frac{x}{2}\right)\left(\frac{k}{x}\right)^5+15\left(\frac{x}{2}\right)^2\left(\frac{k}{x}\right)^4+20\left(\frac{x}{2}\right)^3\left(\frac{k}{x}\right)^3$$

$$+15\left(\frac{x}{2}\right)^4\left(\frac{k}{x}\right)^2+6\left(\frac{x}{2}\right)^5\left(\frac{k}{x}\right)+\left(\frac{x}{2}\right)^6$$

$$20\left(\frac{x}{2}\right)^3\left(\frac{k}{x}\right)^3 = 20$$

$$20\,\frac{1}{8}\,k^3 = 20$$

$$k = 2$$

2. (2 points) Show that in the decimal expansion of the quotient of two integers, eventually some block of digits repeats. (Examples:

$$\frac{1}{6} = 0.1\underline{6}66\ldots, \qquad \frac{217}{660} = 0.32\underline{87}8787\ldots)$$

Hint: If we divide a by b, the remainder is one of 1,2,...,b−1. Consider what happens after b divisions.

$\frac{a}{b}$

k = pi~~g~~geon holes

n = pidgeons

k = b − 1 pigeonholes

n = b pigeons or more

$\frac{a}{b}$ = R $\frac{1 \ldots b-1}{b}$

n > k

so some block of digits repeat

at least b times $\left\{ \begin{array}{l} \frac{1}{b} \\ \frac{2}{b} \\ \vdots \\ \frac{b-1}{b} \end{array} \right.$

3. (3 points) Consider the following recurrence relation and initial conditions:

2

$$U_n = U_{n-1} + 2U_{n-2} + 2n^2 - 10n + 9, \qquad U_1 = 13, \quad U_2 = -12.$$

You may assume that $U_n^{(p)} = -n^2$ is a particular solution to the recurrence relation. Find U_{100}. (No need to simplify your formula.)

$r^2 - r - 2 = 0$

$(r-2)(r+1) = 0$

$r_1 = 2 \quad r_2 = -1$

$U_n^{(h)} = a(2)^n + b(-1)^n$

$13 = 2a - b$

$-12 = 4a + b$

$1 = 6a \quad a = \frac{1}{6}$

$b = -\frac{38}{3}$

$-12 - \frac{4}{6} = b$

$b = -\frac{38}{3}$

$U_n = U_n^{(h)} + U_n^{(p)} = \frac{1}{6}(2)^n - \frac{38}{3}(-1)^n - n^2$

$U_{100} = \frac{1}{6}(2)^{100} - \frac{38}{3} - 1000$

4. (3 points) In each case below, either give an example of a graph satisfying the stated conditions or explain why there is no such example:

1. a connected graph with 2 edges which does not admit an Euler cycle;
2. a simple graph with 4 vertices whose adjacency matrix equals its incidence matrix for some ordering of the vertices and the edges.

1 / 1

1. 

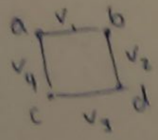
2. 

Fig 1

$$A = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 1 & 0 & 0 & 1 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 1 & 1 & 0 \end{array}$$

B

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & \cancel{0}1 & 1 \\ 0 & 0 & \cancel{0}1 & 1 \end{bmatrix}$$

Not sure what graph these refer to

$$I = \begin{array}{c|cc} & v_1 & v_3 \\ \hline c\ \cancel{a} & 0 & 1 \\ a\ \cancel{b} & 1 & 0 \\ b\ \cancel{c} & 1 & 0 \\ d\ \cancel{d} & 0 & 1 \end{array}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & & 0 \end{bmatrix}$$

0 / 2

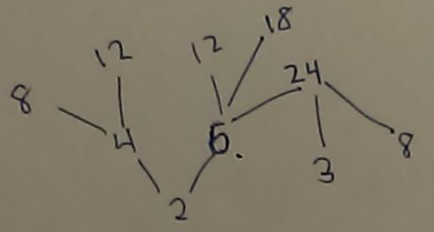
not possible, ~~B would be the only incidence matrix possible~~ the adjacency matrix has to be symmetric with 0s in the diagonals while the incidence matrix can only have 2 1s in each column and no 2 columns can be the same. A matrix with all these properties is not possible

Need to prove this.

5. (3 points) In this problem, we are concerned with the divisibility relation on the set of integers $\{2, 3, 4, \ldots\}$ given by

$$\{(x, y) \mid x \text{ divides } y\}.$$

Consider the associated digraph, and let G be the underlying (undirected) graph, obtained by forgetting that the edges have a direction. Is G connected? What is the shortest path length for two vertices in the same component? *Hint: the answer to the last question will depend on the two vertices picked.*



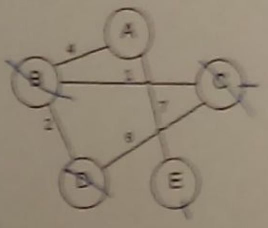
why?

Yes G is connected

Shortest path length is 1 between 2 vertices
eg 2 and 4

misunderstood Q

6. (2 points) Write the order in which the shortest path algorithm visits the vertices of the graph when finding the shortest path from C to E. (Here, we say that the algorithm visits a vertex when the label of the vertex becomes permanent.)



1st run : $v = C$ $P = \{C\}$ $L(D) = \min\{\infty, 6\} = 6$ $L(B) = \min\{\infty, 1\} = 1$

2 : $v = B$ $P = \{B, C\}$ $L(A) = \min\{\infty, 5\} = 5$ $L(D) = \min\{6, 3\} = 3$

3 : $v = D$ $P = \{B, C, D\}$

4 : $v = A$ $P = \{A, B, C, D\}$ $L(E) = \min\{\infty, 12\} = 12$

5 : $v = E$

C B D A E