2. (3 points) Prove that

$$\frac{1}{2}+\frac{2}{3}+\cdots+\frac{n}{n+1}<\frac{n^2}{n+1}$$

for all integers $n \geq 2$.

$S(n): \frac{1}{2}+\frac{2}{3}+\dots+\frac{n}{n+1} < \frac{n^2}{n+1}$

Basic step: $n=2$

$\frac{1}{2}+\frac{2}{3} < \frac{4}{3} \Rightarrow \frac{7}{6} < \frac{8}{6}$ ✓

Inductive step:

$\underbrace{\frac{1}{2}+\frac{2}{3}+\dots+\frac{n}{n+1}}_{S(n)} + \frac{n+1}{n+2} < \underbrace{\frac{n^2}{n+1}}_{S(n)} + \frac{n+1}{n+2}$

inductive hypothesis

Now wts: $\frac{n^2}{n+1} + \frac{n+1}{n+2} < S(n+1) = \frac{(n+1)^2}{n+2}$

$\frac{n^2(n+2)+(n+1)^2}{(n+1)\cancel{(n+2)}} < \frac{(n+1)^2}{\cancel{(n+2)}}$

(sign does flip when mult $n+1$ b/c $n \geq 2 \Rightarrow$ not neg)

$\Rightarrow \frac{n^2(n+2)}{n+1} + (n+1) < n^2+2n+1$

$\Rightarrow \frac{n^2(n+2)}{n+1} < n^2+n$

$\Rightarrow n^2(n+2) < (n^2+n)(n+1)$

$\cancel{n^3} + 2n^2 < \cancel{n^3} + \cancel{n^2} + \cancel{n^2} + n$

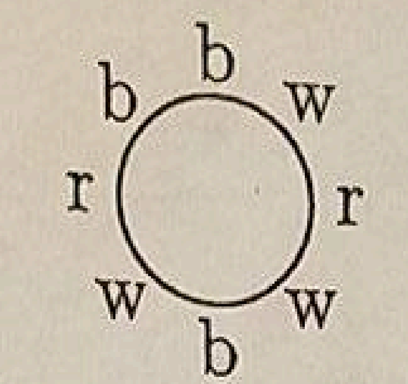
$0 < -n$ since $n \geq 2$ this is always true

P.M.I $S(n)$ is always true for $n \geq 2$

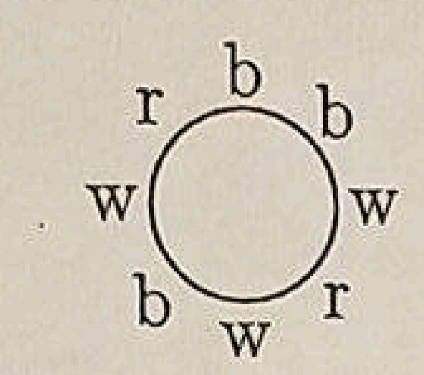
3

3 points) There are 2 red balls, 3 white balls, and 3 blue balls. It is understood that balls of he same color are indistinguishable. The balls are to be arranged in a circle, for example like his:

8 place



(a) In how many ways can they be arranged if we don't distinguish between arrangements obtained from each other by rotation? I.e. we don't distinguish for example between the arrangement above and the following one:



$$\frac{C(8,2)\cdot C(6,3)\cdot C(3,3)}{8}$$

(b) What if we add the condition that the red balls are to be placed next to each other?

group 2 red balls into 1 group ⇒ 1 group red, 3 blue, 3 white

$$\frac{C(7,3)\ C(4,3)\ C(1,1)}{7}$$

3. (2 points) Let s be the sequence

2

$$s_n = \prod_{i=1}^{n} i^{(-1)^i}$$

for all positive integers n. What are s_1, s_2, s_3, s_4? Is s increasing? Decreasing? Nonincreasing? Nondecreasing?

$s_1 = 1^{(-1)^1} = 1^{-1} = +1$

$s_2 = 1^{(-1)^1} \cdot 2^{(-1)^2} = (+1)(2)^1 = +2$

$s_3 = 1^{(-1)^1} \cdot 2^{(-1)^2} \cdot 3^{(-1)^3} = (+2)(\frac{1}{3}) = \frac{2}{3}$ ✓

$s_4 = 1^{(-1)^1} \cdot 2^{(-1)^2} \cdot 3^{(-1)^3} \cdot 4^{(-1)^4} = \frac{2}{3} 4 = \frac{8}{3}$

No, it is not increasing b/c $s_3 < s_2$

No, it is not decreasing b/c $s_4 > s_3$ ✓

No, it is not nonincreasing b/c $s_4 > s_3$

No, it is not nondecreasing b/c $s_3 < s_2$

1-6

5. (2 points) Two dice are rolled, one blue and one red. How many outcomes have either the blue die 3 or an even sum or both?

2

A = # blue die 3 = 1(6) = 6 outcomes

B = # of even sum = 18

$A \cap B = 3$

blue

red	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

=> either blue die 3 or even sum or both = 6 + 18 - 3 = 21

2

1. (2 points) Let A and B be sets. Does the following identity hold?

$$\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$$

If it holds, provide a proof. If it doesn't hold, give a counter-example. (Here, $\mathcal{P}(X)$ denotes the power set of X.)

No, it doesn't

$A = \{0, 1\}$ $B = \{1, 2\}$

$A \cup B = \{0, 1, 2\}$ $P(A \cup B) = \{\{0\}, \emptyset, \{1\}, \{2\}, \{0,1\}, \{$
$\{1,2\}, \{0,1,2\}\}$

$P(A) = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$
$P(B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

$P(A) \cup P(B)$ does not contain $\{0, 1, 2\} \Rightarrow$
$P(A \cup B) \neq P(A) \cup P(B)$

4. (3 points) For this problem recall that a function is a particular type of relation. Let X be a set of n elements ($n \geq 1$). Describe all functions $f : X \to X$ which are also equivalence relations. For each such function, determine the number of equivalence classes.

f is the identity functions $f(x) = x$

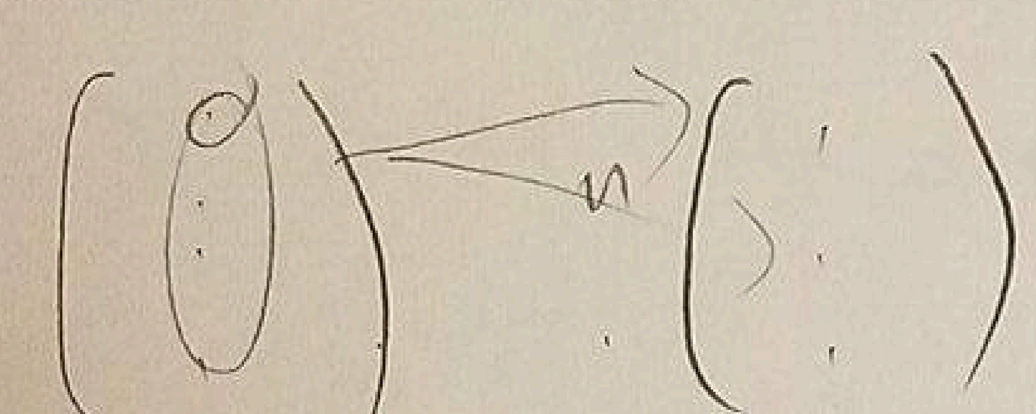
3

it is reflexive because $x' \in X$ there's $f(x) = x \Rightarrow xRx$

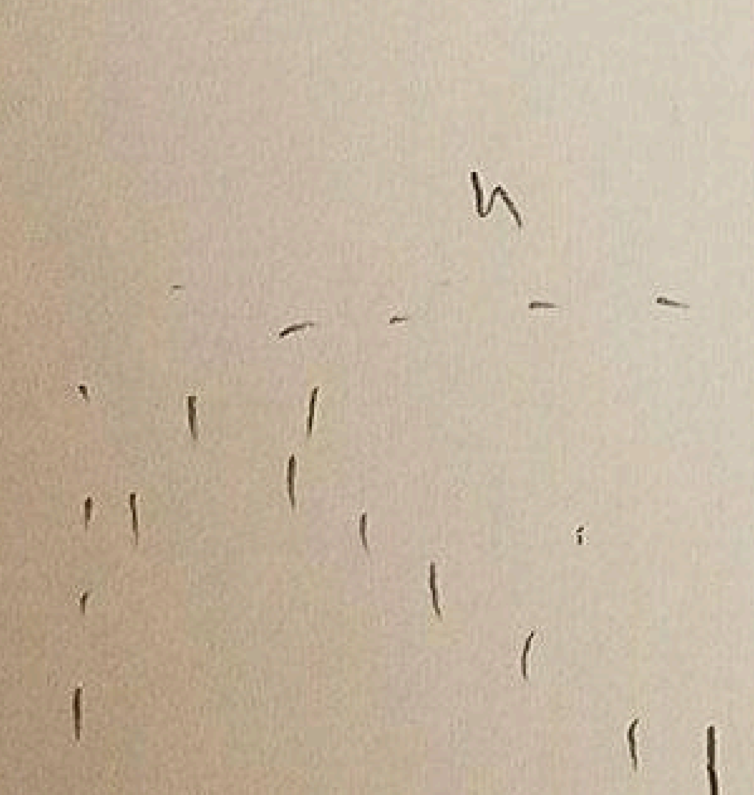
it's symmetric because it's the identity function, every element relates to itself

it's transitive .

the # of equivalen classes is n b/c only x relates to itself and there're n elements,



the identity is the only function that's also a equivalen relation b/c it has to reflexive
$\Rightarrow$ all x_i are taken $\Rightarrow$ can't have other



Math 61
Fall 2017
10/23/17
Time Limit: 50 Minutes

Name (Print): [redacted]
SID Number:

Day \ T.A.	Eric	David	Chris
Tuesday	2A	2D	2F
Thursday	2B	2C	2E

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter your name and SID number on the top of this page, and put your initials on the top of every page, in case the pages become separated. Also, have your photo ID on the desk in front of you during the exam.

Calculators or computers of any kind are not allowed. You are not allowed to consult any other materials of any kind, including books, notes and your neighbors. You may use the back of this sheet for your notes ("scratch paper"). If you need additional paper, let the proctors know.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a result from class, discussion, or homework you must indicate this** and explain why the result may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	2	2
2	3	3
3	2	2
4	3	3
5	2	2
6	3	3
Total:	15	15

Of course, if you have a question about a particular problem, please raise your hand and one of the proctors will come and talk to you.