**Math 61 - Summer 2020 - 07/23/2020**
**Quiz 4**
**Name and ID:**

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Please remember that your work is graded on the quality of your writing and explanation as well as the validity of the arguments and calculations.

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(1) (4 points) Consider *all possible* complete bipartite graphs on 12 vertices (for $K_{m,n}$, $m+n=12$ must hold). What is the maximum number of edges one can achieve? Justify your answer.

Let the number of vertices of one side be $n$
Then the number of vertices of the other side would be $12-n$.

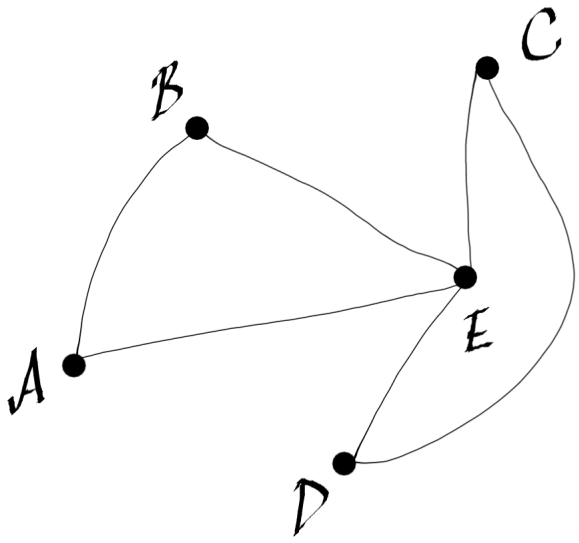
$\because$ total number of edges would be $n\cdot(12-n)$.

$\therefore$ $\frac{d}{dn}\left(n\cdot(12-n)\right) = \frac{d}{dn}\left(12n-n^2\right) = 12-2n. = 0.$

$n = 6.$

$\therefore$ $6\times 6 = 36.$ The maximum number of edges one can achieve is 36.

(2) (6 points) How many paths of length 3 does the following graph have from vertex $E$ to vertex $B$? Justify your answer.




$A =$

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 0 | 1 | 0 | 0 | 1 |
| B | 1 | 0 | 0 | 0 | 1 |
| C | 0 | 0 | 0 | 1 | 1 |
| D | 0 | 0 | 1 | 0 | 1 |
| E | 1 | 1 | 1 | 1 | 0 |

$A^3 =$

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 2 | 3 | 2 | 2 | 5 |
| B | 3 | 2 | 2 | 2 | 5 |
| C | 2 | 2 | 2 | 3 | 5 |
| D | 2 | 2 | 3 | 2 | 5 |
| E | 5 | 5 | 5 | 5 | 4 |

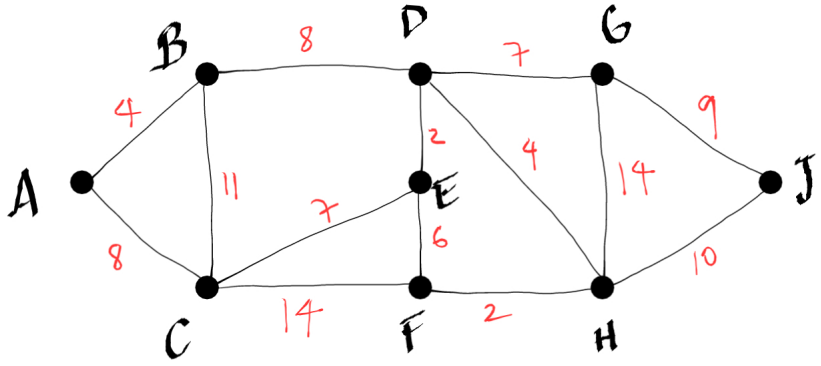
∵ The entry from row E, column B is 5

∴ There are 5 paths from E to B of length 3.

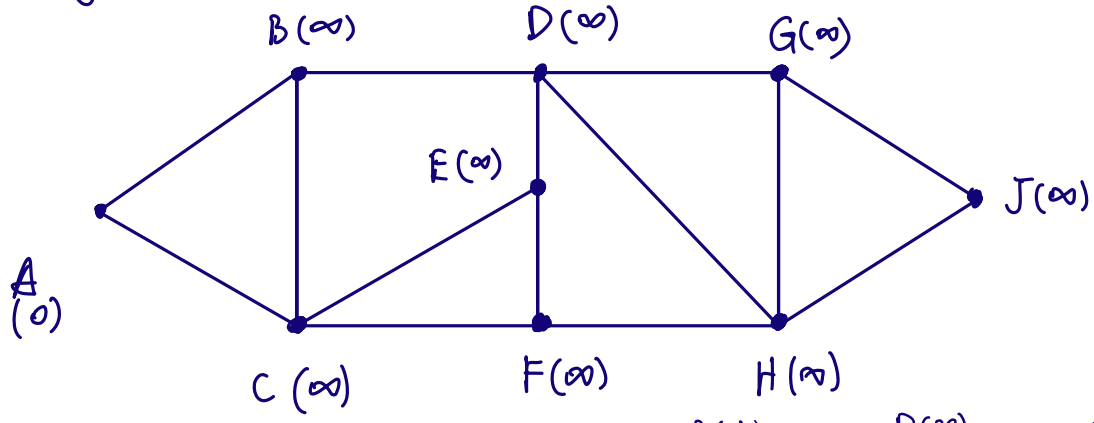
(E, A, E, B)
(E, D, E, B)
(E, C, E, B).
(E, B, E, B).
(E, B, A, B).

(3) (10 points) Find the *shortest path* from vertex $A$ to vertex $J$ in the following graph. (Recall that shortest path is the path with smallest total weight.) You must list the path and its total weight, but you do not need to justify your answer (though, you may want to make sure it is correct using Dijkstra's algorithm).

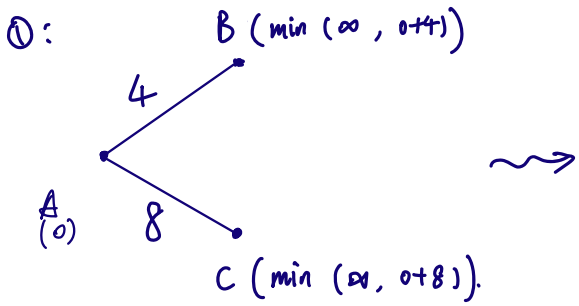
The length of the shortest path from A to J is 26.
The shortest path is: (A, B, D, H, J).

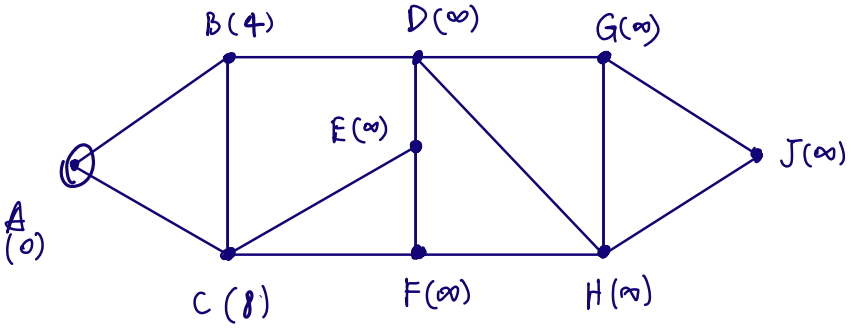



Assign the initial labels:

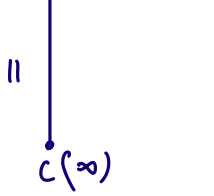


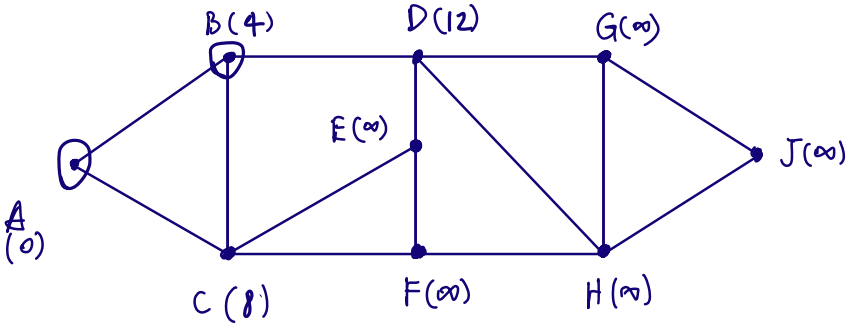

①:



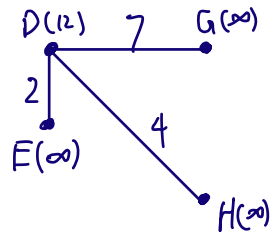



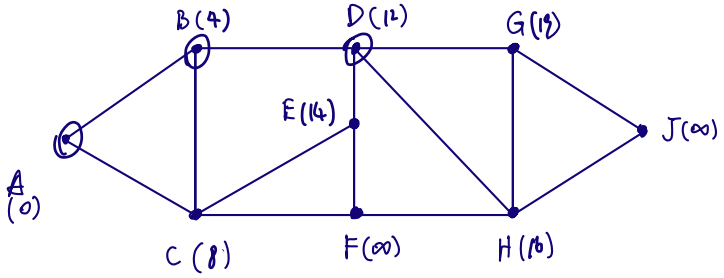

②, B(4) 8 D(∞)



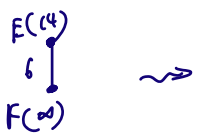



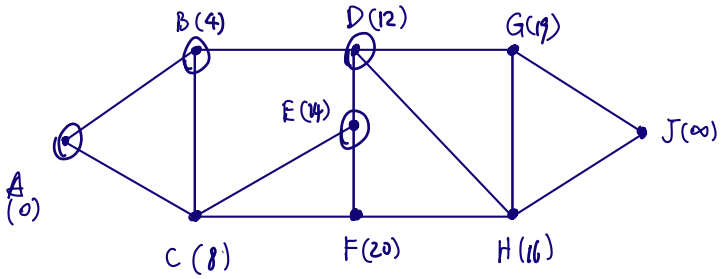
③ :



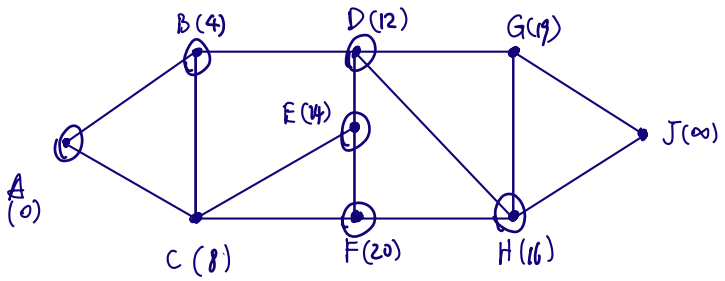




④ :

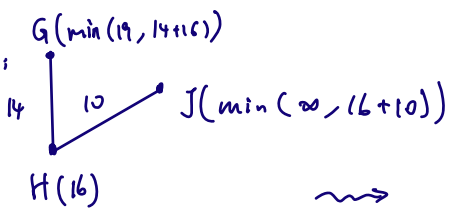


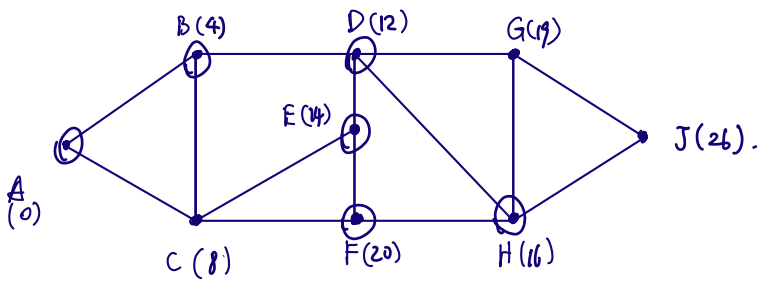




⑤ :

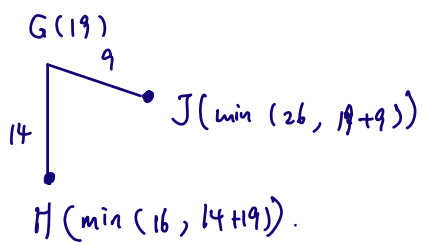



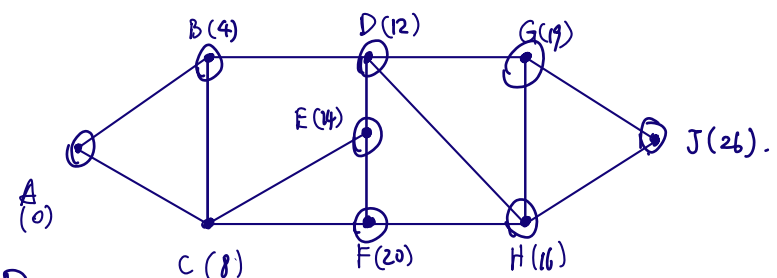

⑥ :




⑦ :




⑧ : We have reached J. Done

∴ L(J) = 26

The shortest path is :

(A, B, D, H, J).