Math 61 - Summer 2020 - 07/16/2020
Quiz 3
Name and ID:

Please remember that your work is graded on the quality of your writing and explanation as well as the validity of the calculations.

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(1) (12 points) Consider the following recurrence relation

$$a_n = 6\,a_{n-1} - 9\,a_{n-2}, \quad \text{for} \quad n \geq 2$$

alongside the initial conditions $a_0 = -1$ and $a_1 = 1$.

(a) (2 pts) Write out the first five terms of the sequence $a = \{a_n\}_{n=0}^{\infty}$

$a_0 = -1$

$a_1 = 1$

$a_2 = 6a_1 - 9a_0 = 6 + 9 = 15$

$a_3 = 6a_2 - 9a_1 = 81$

$a_4 = 6a_3 - 9a_2 = 351$

$\therefore$ The first five terms are $-1, 1, 15, 81, 351$

(b) (10 pts) Find a closed formula (not a recurrence) for the sequence a of the form

$$a_n = f(n).$$

say $a_n = t^n$ $\quad t^n = 6t^{n-1} - 9t^{n-2}$

$t^n - 6t^{n-1} + 9t^{n-2} = 0$

$t^{n-2} \cdot (t^2 - 6t + 9) = 0.$

$t^2 - 6t + 9 = 0$

$(t-3)^2 = 0$

$t_1 = t_2 = 3.$

$\therefore \quad a_n = b\,3^n + d\,n\,3^n$

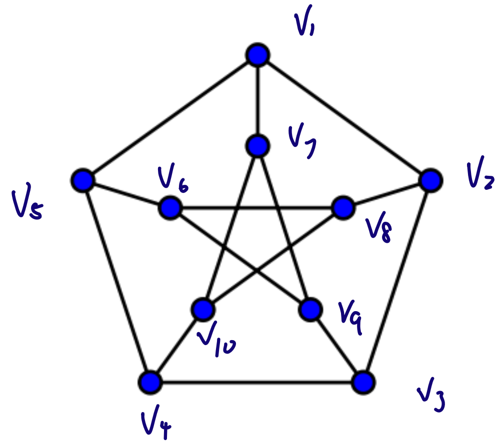
$a_0 = -1 = b \cdot 3^0 + d \cdot 0 \cdot 3^0 = b$

$a_1 = 1 = b \cdot 3 + d \cdot 3 = -3 + 3d$

$\therefore \begin{cases} b = -1 \\ d = \frac{4}{3} \end{cases}$

$\therefore \quad a_n = -3^n + \frac{4}{3} n \cdot 3^n$

(2) (8 points) Consider the following graph, G.



(a) (4 pts) Is G bipartite? Justify your answer.

Suppose G is bipartite:

$\exists V_1, V_2$ s.t. $V_1 \cap V_2 = \emptyset$ & $V_1 \cup V_2 = \{V_1, V_2, \cdots, V_{10}\}$

Say $V_6 \in V_1$, then $V_8 \in V_2$, then $V_2 \in V_1$, then $V_1 \in V_2$,
then $V_5 \in V_1$, then $V_6 \in V_2$

(V_6 is in both V_1 and V_2, which is a contradiction since V_1 and V_2 are disjoint).

$\therefore$ G is not bipartite.

(b) (4 pts) Does this graph have a *simple* cycle of length 6? If YES, you must provide an explicit cycle. If NO, you should briefly explain why.
(HINT: The length of a path is its number of edges.)

Yes,

$(V_1, V_2, V_8, V_{10}, V_4, V_5, V_1)$.

$(V_2, V_3, V_9, V_6, V_5, V_1, V_2)$

$(V_3, V_4, V_{10}, V_7, V_1, V_2, V_3)$

$(V_4, V_5, V_6, V_8, V_2, V_3, V_4)$.

$(V_5, V_1, V_7, V_9, V_3, V_4, V_5)$.