Math 61 - Summer 2020 - 06/25/2020

Quiz 1

Name and ID:

Please remember that your work is graded on the quality of your writing and explanation as well as the validity of the calculations.

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(1) (6 points) Are the propositions "$(p \implies q) \implies r$" and "$p \implies (q \implies r)$" logically equivalent? Justify your answer using a truth table, whatever your answer might be.

p	q	r	$p \to q$	$(p \to q) \to r$	$q \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	F	T	F	T	T
T	T	F	T	F	F	F
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	F	T	T	T	T	T

$\therefore$ $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent

$(x+y)^2 > (x+2y)^2$

$\cancel{x^2} + \cancel{y^2} + 2xy > \cancel{x^2} + \cancel{4}y^2 + 4xy$

$2xy > 3y^2 + 4xy$

$2xy + 3y^2 < 0$

$y \cdot (2x+3y) < 0$

let $x > -\frac{3y}{2}$, $y < 0$

$(x+y)^2 - (x+2y)^2$

$= \cancel{x^2} + 2xy + y^2 - \cancel{x^2} - 4y^2 - 4xy$

$= -3y^2 - 2xy$

$= -y \cdot (3y + 2x)$

$\therefore x > -\frac{3y}{2}$ ... $3y + 2x > 0$

$y > 0$, $2x+3y < 0$. $y>0$ $x>0$ $\therefore$ rejected,

$\boxed{y<0, \quad 2x+3y > 0}$

$2x+3y > 0$

$\boxed{x > -\frac{3y}{2}}$

$\because -y > 0$.

$\therefore -y\cdot(3y+2x) > 0$

$\therefore (x+y)^2 - (x+2y)^2 > 0 \quad \therefore (x+y)^2 > (x+2y)^2$

(2) (7 points) Prove the following proposition if it is true, give a counter-example if it is false

$$\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}, \quad s.t. \quad (x+y)^2 > (x+2y)^2.$$

iff $3y^2 < -2xy$ ■

for $y \in (-\frac{2x}{3}, 0)$, $y^2 < \frac{4x^2}{9}$, $3y^2 = \frac{4x^2}{3}$

$-2xy > -2x\cdot(-\frac{4x}{3})$

$= \frac{4x^2}{3}$

$\therefore -2xy > 3y^2$

■

let $P(x,y)$ denote the expression $(x+y)^2 > (x+2y)^2$.

if the domain of discourse is $\mathbb{R}^+ \times \mathbb{R}$:

let y be an arbitrary element in $\mathbb{R}$:

if $y>0$, $x+y < x+2y$.

$\therefore (x+y)^2 < (x+2y)^2$.

if $y<0$, when $x+y=0$: $(x+y)^2 = 0$

$(x+2y)^2 = y^2$

$\therefore (x+y)^2 < (x+2y)^2$

if $y=0$, $(x+y)^2 = x^2 = (x+2y)^2 = x^2$

counter-example: $x=1$, $y=-1$. $(x+y)^2 = 0$ $\quad 0<1$

$(x+2y)^2 = 1$ $\quad \therefore (x+y)^2 < (x+2y)^2$

$\therefore$ If the domain of discourse is $\mathbb{R}^+ \times \mathbb{R}$,

$\forall x \exists y\, P(x,y)$ is false.

(3) (7 points) Prove, using induction, that

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

let $P(n)$ be the statement : $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

(Basis step): When $n = 1$: $\sum_{i=1}^{1} i^3 = 1 = \left(\frac{1+1}{2}\right)^2 = 1$

$\therefore$ $P(1)$ is true.

(Inductive step): When $n=k$: It is assumed that $\sum_{i=1}^{k} i^3 = \left(\frac{k\cdot(k+1)}{2}\right)^2$

When $n=k+1$:

$$\sum_{i=1}^{k+1} i^3$$

$$= \sum_{i=1}^{k} i^3 + (k+1)^3$$

$$= \left(\frac{k\cdot(k+1)}{2}\right)^2 + (k+1)^2$$

$$= \frac{(k\cdot(k+1))^2 + 2^2\cdot(k+1)^2}{2^2}$$

$$= \frac{(2k+2+k^2+k)^2}{2^2}$$

$$= \frac{(k^2+3k+2)^2}{2^2}$$

$$= \left(\frac{(k+1)\cdot(k+2)}{2}\right)^2$$

$\therefore$ Since, $P(1)$ is true, $P(k+1)$ is true whenever $P(k)$ is true,

it is proven that $\sum_{i=1}^{n} i^3 = \left(\frac{n\cdot(n+1)}{2}\right)^2$