$4x-3=y$

Let $y=6$ $\quad 4x-3=6$

$4x=9$

$x=\frac{9}{4}$ but $\frac{9}{4}\notin\mathbb{Z}$

Midterm 1 (Version A)

UCLA: Math 61, Winter 2018

Instructor: Jens Eberhardt
Date: 02 February 2017

- This exam has 4 questions, for a total of 36 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.

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Discussion section (please circle):

Day/TA	HUNT, CHRISTOPHER	HAN, KYUTAE	MENEZES, DEAN
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

Question	Points	Score
1	12	10
2	10	10
3	6	6
4	8	8
Total:	36	34

Please note! The following two pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)	A			
(b)	A			
(c)			C	
(d)			C	
(e)				D
(f)	A			

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (2 points) Let $X = \{1, 2, 3\}$, then $R = \{(1, 3), (2, 2), (3, 1)\}$ is

A. bijective
B. reflexive
C. antisymmetric
D. not symmetric

(b) (2 points) Let $X = \mathbb{Z}_{\geq 0} = \{0, 1, 2, \ldots\}$. Define an equivalence relation R on X by:

$$xRy \text{ if } x - y \text{ is divisible by 3.}$$

reflexive
transitive
symmetric

Then the partition $\mathcal{P}_R$ associated to the relation R is:

A. $\mathcal{P}_R = \{\{0, 3, 6, \ldots\}, \{1, 4, 7, \ldots\}, \{2, 5, 8, \ldots\}\}$
B. $\mathcal{P}_R = \{\{0, 2, 4, \ldots\}, \{1, 3, 5, \ldots\}\}$
C. $\mathcal{P}_R = \{\{0\}, \{1\}, \{2\}, \ldots\}$
D. $\mathcal{P}_R = \{(0, 0), (0, 3), (3, 0) \ldots, (3, 3), (3, 6), (6, 3), \ldots,$
$(1, 1), (1, 4), (4, 1), \ldots, (4, 4), (4, 7)(7, 4), \ldots,$
$(2, 2), (2, 5), (5, 2), \ldots, (5, 5), (5, 8)(8, 5) \ldots\}$

(1,1), (1,4), (4,1),
(1,7), (7,1)

(0,0) (0,3), (3,0) (3,6), (6,3), (6,0)

2,5 5,2 5,8

(c) (2 points) Define a partition $\mathcal{P}$ on $\{0, 1, 2, 3\}$ by

$$\mathcal{P} = \{\{0\}, \{1, 3\}, \{2\}\}.$$

0,0 1,1 3,3 2,2
1,3 3,1

Let $R_\mathcal{P}$ be the associated equivalence relation on $\{0, 1, 2, 3\}$. Then

A. $R_\mathcal{P} = \{(0, 0), (1, 3), (3, 1), (2, 2)\}$
B. $R_\mathcal{P} = \{\{0\}, \{1, 3\}, \{2\}\}$
C. $R_\mathcal{P} = \{(0, 0), (1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$
D. $R_\mathcal{P} = \{0, 1, 2, 3\}$

(d) (2 points) Let $X = \{0, 1, 2, 3\}$. For a set Y denote by $\mathcal{P}(Y) = \{S | S \text{ is a subset of } Y\}$ the power set of X. Then

A. $|\mathcal{P}(X \times X)| = 2 \cdot 16$
B. $|\mathcal{P}(X \times X)| = 2^8$
C. $|\mathcal{P}(X \times X)| = 2^{16}$
D. $|\mathcal{P}(X \times X)| = 2 \cdot 8$

2^{n^2}

(e) (2 points) Let $n \geq 1$ be a positive integer. Then

$$\prod_{i=1}^{n} 2i$$

1 2 3
2 8 48
$2i$ $2 \cdot 4 \cdot 6 \cdot 8$

is equal to

A. $2^n \frac{n(n+1)}{2}$
B. $\frac{2^n 2^{n+1}}{2}$
C. $(2n)!$
D. $2^n \cdot n!$

$2^n \cdot n!$

(f) (2 points) Let $X = \{a, b\}$. Denote by $X^{\geq 3}$ the set of all strings over X of length bigger or equal than three and by X^* the set of all strings over X. Then

A. $|X^* - X^{\geq 3}| = 5$
B. $|X^* - X^{\geq 3}| = 6$
C. $|X^* - X^{\geq 3}| = 7$
D. $|X^* - X^{\geq 3}| = 8$

ab aa
ba bb
a
b
λ

2. In the following questions, simply write down your answer. There is *no justification needed.* Do not simplify expressions as $2^4, 6!, C(n, r), \ldots$.

(a) (2 points) Determine the number of 7-bit strings starting in 1010.

1010 2·2·2

2^3

(b) (2 points) Determine the number of 7-bit strings ending in 1010.

2 2 2 1010

2^3

(c) (2 points) Determine the number of 7-bit strings starting in 1010 or ending in 1010.

No overlap
1010_ _ _
_ _ _1010

$2^3 + 2^3 - 0 =$ 16

2^4

(d) (2 points) You have three friends: Rocco, Gina and Hans. And you have seven different sweets: a popsicle, a piece of apple pie, a chocolate bar, a Berliner, a jelly doughnut, a marshmallow and a lemon drop.
You want to give two sweets to Rocco, three to Gina and two to Hans. In how many ways could you do this?

distinct, no repetition ordered

$C(7,2) \cdot C(5,3) \cdot C(2,2)$

(e) (2 points) Let $X = \{1, 2, 3, 4, 5\}$. Determine the number of elements of the following set

$$\{S \mid S \text{ is a subset of } X \text{ and } |S| = 3\}.$$

$C(5,3)$

3. (6 points) Prove by induction that

$$\sum_{i=1}^{n}(2i - n) = n$$

for any integer $n \geq 1$.

Hint: Use

$$\sum_{i=1}^{n+1}(2i - (n+1)) = \left(\sum_{i=1}^{n+1}(2i - n)\right) - (n+1).$$

Basis Step: Let $n=1$. $\sum_{i=1}^{1}(2i-n) = 2-1 = 1 = n$ $\quad 1=1$, $n=1$ //

✓

Inductive Step: Assume $\sum_{i=1}^{n}(2i-n) = n$

$$\sum_{i=1}^{n+1}(2i-(n+1)) = \left(\sum_{i=1}^{n+1}(2i-n)\right) - (n+1)$$

$$= \left(\sum_{i=1}^{n}(2i-n)\right) + (2n+2-n) - (n+1)$$

$$= n + (n+2) - (n+1) = n + \cancel{n} - \cancel{n} + 2 - 1$$

$$\sum_{i=1}^{n+1}(2i-(n+1)) = n+1 \quad //$$

✓

By mathematical induction, we conclude that $\sum_{i=1}^{n}(2i-n) = n$ for all $n \geq 1$. ∎

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4. Answer the following questions, *justifying* your answers. (If an answer is Yes, explain why. If an answer is No, give a counterexample.)

(a) (2 points) Define $f : \mathbb{R} \to \mathbb{R}_{\geq 0}, x \mapsto x^2$. Is f surjective?

Yes f is surjective. For every $y \in \mathbb{R}_{\geq 0}$, there exists a corresponding $x \in \mathbb{R}$ such that $x^2 = y$.
Let $m \in \mathbb{R}_{\geq 0}$. $m = x^2$ for all $x \in \mathbb{R}$.

(b) (2 points) Let X, Y, Z be sets. Then

$$X \cap (Y \cup Z) = (X \cap Y) \cup Z.$$

No. Let $X = \{1,2\}$, $Y = \{2,3\}$, and $Z = \{4,5\}$.

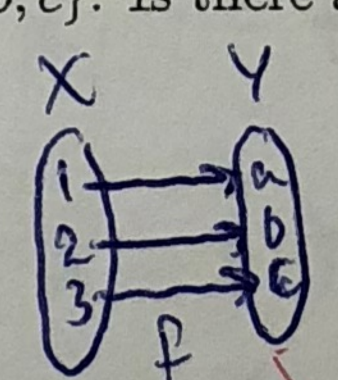
$X \cap (Y \cup Z) = \{1,2\} \cap (\{2,3\} \cup \{4,5\}) = \{1,2\} \cap \{2,3,4,5\} = \{2\}$

$(X \cap Y) \cup Z = (\{1,2\} \cap \{2,3\}) \cup \{4,5\} = \{2\} \cup \{4,5\} = \{2,4,5\}$

$\{2\} \neq \{2,4,5\}$. Therefore $X \cap (Y \cup Z) \neq (X \cap Y) \cup Z$.

(c) (2 points) Let $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$. Is there a bijective function $f : X \to Y$?

Yes. Let $f : X \to Y$.



f is one-to-one since for each $x \in X$, there is a unique $y \in Y$ such that if $f(x_1) = f(x_2)$, $x_1 = x_2$. f is onto since for every $y \in Y$, there exists a corresponding $x \in X$. Since f is one-to-one and onto, f is a bijection.

If $a = a$, $1 = 1$. ✓
If $b = b$, $2 = 2$. ✓
If $c = c$, $3 = 3$. ✓

$f(1) = a$
$f(2) = b$
$f(3) = c$

(d) (2 points) Let $X = \{a, b, c, \ldots, z\}$ be the alphabet. Let α and β be strings over X. Is

$$\alpha\beta = \beta\alpha?$$

No. Let $a = \alpha$ and $b = \beta$. $ab \neq ba$ because strings are sequences which take order into account. A string requires order unlike sets.