Midterm 1 (Version A)

UCLA: Math 61, Winter 2018

Instructor: Jens Eberhardt
Date: 02 February 2017

- This exam has 4 questions, for a total of 36 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.

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Discussion section (please circle):

Day/TA	HUNT, CHRISTOPHER	HAN, KYUTAE	MENEZES, DEAN
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

Question	Points	Score
1	12	12
2	10	10
3	6	3
4	8	8
Total:	36	33

Please note! The following two pages will not be graded. You must indicate your answers **here** *for them to be graded!*

Question 1.

Part	A	B	C	D
(a)	X			
(b)	X			
(c)			X	
(d)			X	
(e)				X
(f)			X	

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) Let $X = \{1, 2, 3\}$, then $R = \{(1, 3), (2, 2), (3, 1)\}$ is

   A. bijective
   B. reflexive
   C. antisymmetric
   D. not symmetric

   (b) (2 points) Let $X = \mathbb{Z}_{\geq 0} = \{0, 1, 2, \dots\}$. Define an equivalence relation R on X by:

   $$xRy \text{ if } x - y \text{ is divisible by } 3.$$

   Then the partition $\mathcal{P}_R$ associated to the relation R is:

   A. $\mathcal{P}_R = \{\{0, 3, 6, \dots\}, \{1, 4, 7, \dots\}, \{2, 5, 8, \dots\}\}$
   B. $\mathcal{P}_R = \{\{0, 2, 4, \dots\}, \{1, 3, 5, \dots\}\}$
   C. $\mathcal{P}_R = \{\{0\}, \{1\}, \{2\}, \dots\}$
   D. $\mathcal{P}_R = \{(0, 0), (0, 3), (3, 0) \dots, (3, 3), (3, 6), (6, 3), \dots,$
   $(1, 1), (1, 4), (4, 1), \dots, (4, 4), (4, 7)(7, 4), \dots,$
   $(2, 2), (2, 5), (5, 2), \dots, (5, 5), (5, 8)(8, 5) \dots\}$

   (c) (2 points) Define a partition $\mathcal{P}$ on $\{0, 1, 2, 3\}$ by

   $$\mathcal{P} = \{\{0\}, \{1, 3\}, \{2\}\}.$$

   Let $R_{\mathcal{P}}$ be the associated equivalence relation on $\{0, 1, 2, 3\}$. Then

   A. $R_{\mathcal{P}} = \{(0, 0), (1, 3), (3, 1), (2, 2)\}$
   B. $R_{\mathcal{P}} = \{\{0\}, \{1, 3\}, \{2\}\}$
   C. $R_{\mathcal{P}} = \{(0, 0), (1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$
   D. $R_{\mathcal{P}} = \{0, 1, 2, 3\}$

(d) (2 points) Let $X = \{0,1,2,3\}$. For a set Y denote by $\mathcal{P}(Y) = \{S | S \text{ is a subset of } Y\}$ the power set of X. Then

A. $|\mathcal{P}(X \times X)| = 2 \cdot 16$
B. $|\mathcal{P}(X \times X)| = 2^8$
C. $|\mathcal{P}(X \times X)| = 2^{16}$
D. $|\mathcal{P}(X \times X)| = 2 \cdot 8$

$|(X \times X)| = 16$

$|P(X \times X)| = 2^{16}$

(e) (2 points) Let $n \geq 1$ be a positive integer. Then

$$\prod_{i=1}^{n} 2i$$

is equal to

A. $2^n \frac{n(n+1)}{2}$
B. $\frac{2^n 2^{n+1}}{2}$
C. $(2n)!$
D. $2^n \cdot n!$

(f) (2 points) Let $X = \{a, b\}$. Denote by $X^{\geq 3}$ the set of all strings over X of length bigger or equal than three and by X^* the set of all strings over X. Then

A. $|X^* - X^{\geq 3}| = 5$
B. $|X^* - X^{\geq 3}| = 6$
C. $|X^* - X^{\geq 3}| = 7$
D. $|X^* - X^{\geq 3}| = 8$

$X^* - X^{\geq 3} = X^{0:2}$

$|X^{0:2}| = 7$

$X^{0:2} = \{\lambda, a, b, aa, ab, ba, bb\}$

2. In the following questions, simply write down your answer. There is *no justification needed.* Do not simplify expressions as $2^4, 6!, C(n,r), \ldots$.

(a) (2 points) Determine the number of 7-bit strings starting in 1010.

2^3

(b) (2 points) Determine the number of 7-bit strings ending in 1010.

2^3

(c) (2 points) Determine the number of 7-bit strings starting in 1010 or ending in 1010.

$2^3 + 2^3$

(d) (2 points) You have three friends: Rocco, Gina and Hans. And you have seven different sweets: a popsicle, a piece of apple pie, a chocolate bar, a Berliner, a jelly doughnut, a marshmallow and a lemon drop.
You want to give two sweets to Rocco, three to Gina and two to Hans. In how many ways could you do this?

$s_1 \; s_2 \; R \; s_3 \; s_4 \; s_5 \; G \; s_6 \; s_7$

total orderings of sweets: $7!$

redundant: $2!\,3!\,2!$

$$\boxed{\frac{7!}{2!\,3!\,2!}}$$

(e) (2 points) Let $X = \{1, 2, 3, 4, 5\}$. Determine the number of elements of the following set

$$\{S \mid S \text{ is a subset of } X \text{ and } |S| = 3\}.$$

$\binom{5}{3}$

3. (6 points) Prove by induction that

$$\sum_{i=1}^{n}(2i-n)=n$$

for any integer $n \geq 1$.
Hint: Use

$$\sum_{i=1}^{n+1}(2i-(n+1))=\left(\sum_{i=1}^{n+1}(2i-n)\right)-(n+1). \quad (*)$$

Base Case: $(n=1)$

$$\sum_{i=1}^{1}(2i-1) = 2(1)-1 = 1 \checkmark$$

$\checkmark$

Induction step:

assume $\sum_{i=1}^{n}(2i-n)=n$. Then:

$$\sum_{i=1}^{n+1}(2i-(n+1)) \underset{\text{by } (*)}{=} \left[\sum_{i=1}^{n+1}(2i-n)\right]-(n+1) \underset{\text{by assumption}}{=} n-(n+1) = 1$$

No

$\therefore$ Since our statement is true for $n=1$, and the statement being true for n implies it is also true for $n+1$, by induction, the statement is true for all $n \geq 1$, $n \in \mathbb{Z}$.

4. Answer the following questions, *justifying* your answers. (If an answer is Yes, explain why. If an answer is No, give a counterexample.)

(a) (2 points) Define $f : \mathbb{R} \to \mathbb{R}_{\geq 0}, x \mapsto x^2$. Is f surjective?

Yes. $\forall y \in \mathbb{R}_{\geq 0}$, $\exists x \in \mathbb{R}$ s.t. $x^2 = y$.

(take a square root)

(b) (2 points) Let X, Y, Z be sets. Then

$$X \cap (Y \cup Z) = (X \cap Y) \cup Z.$$

no. let $X = \emptyset$, $Y = \{1\}$, $Z = \{1\}$

$X \cap (Y \cup Z) = \emptyset$

$(X \cap Y) \cup Z = \{1\}$, $\emptyset \neq \{1\}$

(c) (2 points) Let $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$. Is there a bijective function $f : X \to Y$?

Yes. for example, let $f = \{(1, a), (2, b), (3, c)\}$

(d) (2 points) Let $X = \{a, b, c, \ldots, z\}$ be the alphabet. Let α and β be strings over X. Is

$$\alpha\beta = \beta\alpha?$$

no. let $\alpha = a$, $\beta = b$

$\alpha\beta = ab$

$\beta\alpha = ba$, $ab \neq ba$

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