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Section letter: 1G (Chris Hunt Tuesday)

Math 61 : Discrete Structures
Midterm 1
Instructor: Ciprian Manolescu

You have 50 minutes.

No notes, books or calculators are allowed.
Do not use your own scratch paper.

1. (2 points each) **Multiple choice:** Circle the right answer. You do NOT need to justify your answers.

If α is a string of length two, what is the number of substrings of α?

(A) 2; (B) 3; (C) 4; (D) 5; (E) It depends on the string.

In how many ways can we distribute 12 (identical) copies of the same book to 5 people: Alice, Bob, Casey, David and Emily?

(A) 12^5; (B) 5^{12}; (C) $C(12,5)$; (D) $C(16,4)$; (E) $C(16,11)$.

In how many ways can we distribute 12 distinct books to 5 people: Alice, Bob, Casey, David and Emily?

(A) 12^5; (B) 5^{12}; (C) $C(12,5)$; (D) $C(16,4)$; (E) $C(16,11)$.

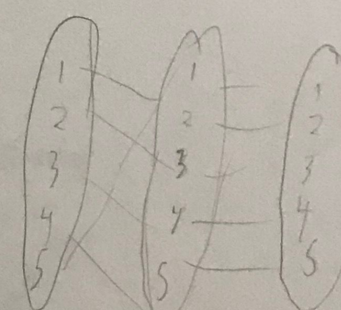
Let $X = \{1, 2, 3, \dots\}$ be the set of all natural numbers. If $f, g : X \to X$ are two functions such that $f \circ g$ is bijective, then:

(A) f has to be bijective, but g does not have to be bijective;

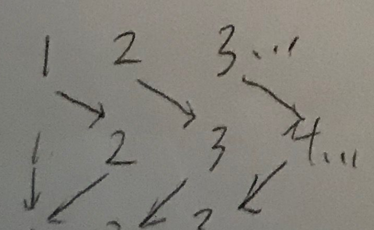
(B) g has to be bijective, but f does not have to be bijective;

(C) Both f and g have to be bijective;

(D) Neither f nor g have to be bijective.



If X is finite (it isn't), both are bijective



2. Write down the answer to each question. *You do NOT need to justify your answers.* Also, you do not need to simplify expressions such as 2^6, $6!$, $C(6,3)$, etc.

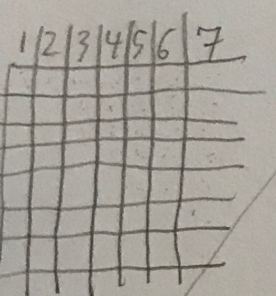
(a) Consider the set $X = \{1, 2, \ldots, 7\}$.

(2 points) How many of the relations on X are NOT reflexive?

$2^{49-7} = \boxed{2^{42}}$ ×

$2^{49} - 2^{42}$

(2 points) How many of the relations on X are symmetric?



$2^{n(n-1)/2}$ $\quad \frac{(n-1)(n)}{2} = \frac{6(7)}{2} = \frac{42}{2} = 21$

$\boxed{2^{28}}$ ✓

(2 points) How many of the relations on X are symmetric OR reflexive (i.e. either symmetric, or reflexive, or both)?

$\boxed{(2^{49} - 2^{42}) + 2^{28} - 2^{7}}$ ×

$2^{28} + 2^{42} - (2^{7})$

(2 points) How many of the relations on X are symmetric but NOT reflexive?

$\boxed{2^{21}}$ ✗

$2^{28} - 2^{21}$

(2 points) How many of the relations on X are both symmetric AND antisymmetric?

$\boxed{2^{49} - 2^{42}}$ ×

2^{28}

2^{7}

(b) (2 points) How many distinct strings can be obtained from the string $AAAABCCDD$ by permuting (re-ordering) its letters?

$\boxed{\dfrac{9!}{4!2!2!}}$ ✓

3. Let $\mathbb{Z}$ be the set of all integers, and $\mathcal{P}(\mathbb{Z})$ the power set of $\mathbb{Z}$ (consisting of all subsets of $\mathbb{Z}$). Consider the following relation R on $\mathcal{P}(\mathbb{Z})$:

$$(A, B) \in R \iff A \cap B = \emptyset.$$

Answer the following questions, fully justifying your answers. (If an answer is *YES*, explain why. If an answer is *NO*, give a counterexample.)

(a) (2 points) Is R reflexive?

$\boxed{\text{No}}$, because if $A = \{1,2,3\}$, $(A,A) \notin R$ because $A \cap A = A = \{1,2,3\} \neq \emptyset$

(b) (2 points) Is R symmetric?

$\boxed{\text{Yes}}$, because $A \cap B = B \cap A$

(c) (3 points) Is R transitive?

$\boxed{\text{No}}$. If $A = \{1,2,3\}$, $B = \{4,5,6\}$, and $C = \{3,7,9\}$, then $(A,B) \in R$ and $(B,C) \in R$ but $(A,C) \notin R$ because $A \cap B = B \cap C = \emptyset$ but $A \cap C = \{3\}$

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(d) (3 points) Prove that $(A, B) \in R \circ R$ for all A and B.

$(A,B) \in R \circ R$ if $A \cap (A \cap B) = \emptyset$ and $B \cap (A \cap B) = \emptyset$?

....close...? $(A \cap B)$ has to equal $\emptyset$ because that determines whether A & B are in R, and any set intersected with $\emptyset$ is $\emptyset$, so $(A,B) \in R \circ R$ for all A & B.

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4. (10 points) Prove by induction on n that:

$$2^{2n} \leq (n+2)!,$$

for any integer $n \geq 1$.

Base case $(S(1))$: $2^{2(1)} = 4$, $(1+2)! = 3! = 6$

$4 \leq 6$, so the base case is valid

Inductive case:

Suppose that $2^{2n} \leq (n+2)!$

Then for $S(n+1)$, $2^{2(n+1)} \leq ((n+1)+2)!$

$2^{2n+2} \leq (n+3)!$

$(2^{2n})(2^2) \leq (n+3)(n+2)!$

$(2^{2n})(4) \leq (n+3)(n+2)!$

We suppose that $(2^{2n}) \leq (n+2)!$, for any integer $n \geq 1$,

and $4 \leq n+3$ is true for any integer $n \geq 1$ as well,

so therefore $(2^{2n})(4) \leq (n+3)(n+2)!$ for any integer $n \geq 1$.

The inductive case is valid.

Do not write on this page.

1	2	out of 8 points
2	4	out of 12 points
3	8	out of 10 points
4	10	out of 10 points
Total	24	out of 40 points