Exam 1

Math 61, Winter 2022

Name: ______________________________ Student ID Number: ____________

Section Number: ______________________________

Read the following instructions before beginning the exam:

- You must submit your exam answers on Gradescope by 8am on Saturday, January 29th.
- You are permitted to look at the textbook, the course website, any notes you've prepared, and standard reference sites (e.g. Wikipedia) while working on the exam.
- You must work alone and submit your own work. You are not allowed to seek any assistance on the exam, whether it be in person or online. In particular, you are **not** permitted to use human resources (including Chegg, Math Stack Exchange, etc.) or to collaborate with anyone. Violations of these rules will be regarded as academic dishonesty and will be reported to the Dean of Students.
- If you have questions during the exam, then you can come to the lecture Zoom link 9-9:50 or 2-2:50. Or you can make a private post to the TAs and instructor on Campuswire.
- You can print this exam and write your answers on it, you can download the exam and write the answers on a tablet, you can use this LaTeX template to type your answers, or you can write your answers cleanly on paper.
- You should give full sentence explanations for Problems 2, 4, and 6. You do not need to write in sentences in Problem 3 or 5.
- There are 80 total points on the exam, 66 points on the written portion, and 14 points on the online portion. Don't forget to do the online portion on Gradescope!

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1. If you are writing on clean paper, then write out the sentence

   I, [INSERT YOUR FULL NAME HERE], have read the directions and have not used any non-permitted materials nor received any help on this exam.

   Make sure to put your name where it says "insert your name here". You must have this sentence with your name. If it is missing, the exam will not be graded.

   If you are printing/downloading and writing on this exam, then write your full name in the box below to indicate you have read and understood the directions:

   I, [______________________]*, have read the directions and have not used any non-permitted materials nor received any help on this exam.*

   Your name must be present in the box above. Otherwise your exam will not be graded.

2. Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function defined by $f(b) = \left\lfloor \frac{b+1}{7} \right\rfloor$. (*Recall for a real number x, $\lfloor x \rfloor = x$ if x is an integer, while $\lfloor x \rfloor$ is the largest integer less than x if x is not an integer, e.g. $\lfloor 1.345 \rfloor = 1$ while $\lfloor -17 \rfloor = -17$.*)

   (a) [10 pts] Is f one-to-one? Justify your answer with a proof.

   (b) [10 pts] Is f onto? Justify your answer with a proof.

You do not need to write full sentences for 3(a) and 3(b). Still make sure to show your work, and make it clear what you are doing. Put your final answer in the box in completely simplified form. You can use a calculator or online calculator to get help you similplify.

3. A pet shelter has a kitten room with 13 distinct kittens. Of the 13, four of them are orange, four of them are gray, four of them are black, and one of them is calico.

   (a) [10 pts] You decide you want to take home 5 kittens, and you want two kittens with one fur color, two kittens with another fur color, and a single kitten of a third color (e.g. two distinct orange, two distinct gray, and one black). How many ways can you choose the 5 kittens?

   Put your final answer in this box:

   (b) [10 pts] You end up deciding to get 2 orange kittens, 2 black kittens, and 1 calico kitten. You get home and decide to take a picture of the kittens sitting in a row, but the orange kittens keep fighting each other. How many ways can you line up the kittens so that the orange kittens are not next to each other? (Individual kittens are still considered distinct.)

   Put your final answer in this box:

4. [10 pts] Let R and S be relations on a set X. Prove that if R is antisymmetric, then $R \cap S$ is antisymmetric. Then provide a counterexample to show that "if R is symmetric, then $R \cap S$ is symmetric" is a false statement.

5. [6 pts] Let $X = \{1, 2, 3, 4, 5\}$. Give an example of an equivalence relation R on X such that $(1, 3) \in R$, $(2, 3) \in R$, and R has at least two distinct equivalence classes. Then write down the partition of X given by the equivalence classes of your example. How many equivalence classes are there?

6. [10 pts] Use the Principle of Mathematical Induction to prove that $C(2n, n) < 2^{2n}$ for all $n \geq 1$.