

# Math 61 Homework Exam 1 Template

Sample Student

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## Problem 1

## Problem 2

Let  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  be the function defined by  $f(b) = \left\lfloor \frac{b+1}{7} \right\rfloor$ .

(a) Is  $f$  one-to-one? Justify your answer with a proof.

*Proof.* □

(b) Is  $f$  onto? Justify your answer with a proof.

*Proof.* □

## Problem 3

A pet shelter has a kitten room with 13 distinct kittens. Of the 13, four of them are orange, four of them are gray, four of them are black, and one of them is calico.

(a) You decide you want to take home 5 kittens, and you want two kittens with one fur color, two kittens with another fur color, and a single kitten of a third color (e.g. two distinct orange, two distinct gray, and one black). How many ways can you choose the 5 kittens?

*Solution.* □

(b) You end up deciding to get 2 orange kittens, 2 black kittens, and 1 calico kitten. You get home and decide to take a picture of the kittens sitting in a row, but the orange kittens keep fighting each other. How many ways can you line up the kittens so that the orange kittens are not next to each other? (Individual kittens are still considered distinct.)

*Solution.* □

## Problem 4

Let  $R$  and  $S$  be relations on a set  $X$ . Prove that if  $R$  is antisymmetric, then  $R \cap S$  is antisymmetric. Then provide a counterexample to show that “if  $R$  is symmetric, then  $R \cap S$  is symmetric” is a false statement.

*Proof.* □

## Problem 5

Let  $X = \{1, 2, 3, 4, 5\}$ . Give an example of an equivalence relation  $R$  on  $X$  such that  $(1, 3) \in R$ ,  $(2, 3) \in R$ , and  $R$  has at least two distinct equivalence classes. Then write down the partition given by the equivalence classes of your example. How many equivalence classes are there?

*Solution.* □

**Problem 6**

Use the Principle of Mathematical Induction to prove that  $C(2n, n) < 2^{2n}$  for all  $n \geq 1$ .

*Proof.*

□