Final Exam: Written Portion

Math 61, Winter 2022

Name: ______________________________ Student ID Number: ____________________

Circle your discussion section:

1A: Kedar Karhadkar, Tues 9am	1C: Yan Tao, Tues 9am	1E: Ben Spitz, Tues 9am
1B: Kedar Karhadkar, Thurs 9am	1D: Yan Tao, Thurs 9am	1F: Ben Spitz, Thurs 9am
2A: Jung Joo Suh, Tues 2pm	2C: Alex Tenenbaum, Tues 2pm	2E: Olha Shevchenko, Tues 2pm
2B: Jung Joo Suh, Thurs 2pm	2D: Alex Tenenbaum, Thurs 2pm	2F: Olha Shevchenko, Thurs 2pm

Read the following instructions before beginning the exam:

- You must submit your written exam answers on Gradescope by 8am on Monday, March 14th.
- You are permitted to look at the textbook, the course website, any notes you've prepared, and standard reference sites (e.g. Wikipedia) while working on the exam.
- You must work alone and submit your own work. You are not allowed to seek any assistance on the exam, whether it be in person or online. In particular, you are **not** permitted to use human resources (including Chegg, Math Stack Exchange, etc.) or to collaborate with anyone. Violations of these rules will be regarded as academic dishonesty and will be reported to the Dean of Students.
- Do not post the exam questions online at any point during or after the exam.
- If you have questions during the exam, then you can come to the lecture Zoom link 11:30am-12:30pm or 4:30pm-5:30pm, or you can make a private post to the TAs and instructor on Campuswire.
- You can print this exam and write your answers on it, you can download the exam and write the answers on a tablet, you can use this LaTeX template to type your answers, or you can write your answers cleanly on paper.
- You must write in full sentences and justify your work for every problem except Problems 2(a) and 3(b).
- There are 105 total points on the final exam. These are distributed across the different parts as shown below:

Exam Portion	Points Available
Final Exam: Online Portion, Part 1	18 points
Final Exam: Online Portion, Part 2	16 points
Final Exam: Online Portion, Part 3	16 points
Final Exam: Written Portion	55 points
Final Exam	**105 total points**

If you are writing on clean paper, then write out the sentence

I, [INSERT YOUR FULL NAME HERE], have read the directions and have not used any non-permitted materials nor received any help on this exam.

Make sure to put your name where it says "insert your name here". You must have this sentence with your name. If it is missing, the exam will not be graded.

If you are printing/downloading and writing on this exam, then write your full name in the box below to indicate you have read and understood the directions:

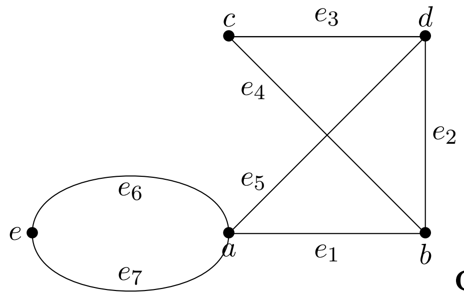
I, ______________________________, *have read the directions and have not used any non-permitted materials nor received any help on this exam.*

Your name must be present in the box above. Otherwise your exam will not be graded.

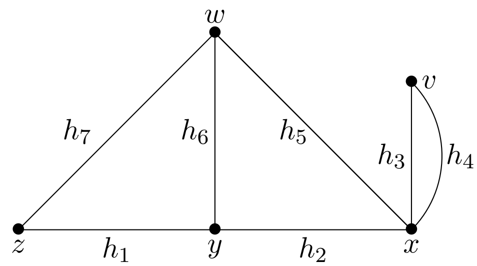
1. [8 pts] Let $\mathcal{S}$ denote the set of all finite subsets of $\mathbb{Z}$ and let $\mathbb{Z}^{\geq 0}$ denote the set of all nonnegative integers. Define a function $f : \mathcal{S} \to \mathbb{Z}^{\geq 0}$ by $f(X) = |X|$. Prove that f is onto.

2. Use the given graphs to answer the questions.

   (a) [5 pts] The graphs G_1 and G_2 shown below are isomorphic. Write down an isomorphism from G_1 to G_2. You do not have to justify your work in this problem; just write down an isomorphism.

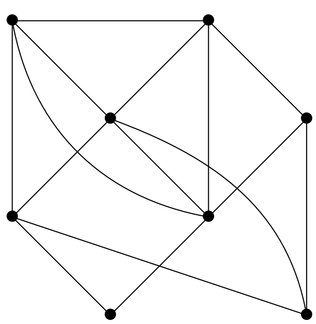


G1



G2

   (b) [5 pts] Determine whether or not the following graph is planar. If the graph is planar, redraw it so that no edges cross; otherwise, find a subgraph homeomorphic to either K_5 or $K_{3,3}$.



3. (a) [9 pts] Let $X = \{1, 2, 3, 4, 5, 6, 7\}$. Define a relation R on $X \times X$ by $(a, b)R(c, d)$ if $a - c = b - d$. Prove that R is an equivalence relation.

   (b) [3 pts] List all elements in $[(2, 1)]$. You do not have to justify your answer for this part; just list the elements.

4. For parts (a)–(c), determine if the following statements are true or false. If the statement is true, explain why. If the statement is false, give a counterexample.

   (a) [5 pts] Let P be an invariant. If G_1 and G_2 are two graphs that both satisfy P, then G_1 and G_2 are isomorphic.

   (b) [5 pts] For all positive integers n, r with $n \geq r$, we have that $P(n, r) \geq C(n, r)$.

   (c) [5 pts] Let G_1 and G_2 be two graphs. If G_1 and G_2 are homeomorphic, then G_1 and G_2 are isomorphic.

5. [10 pts] Consider the sequence c_0, c_1, c_2, $\ldots$ defined by $c_0 = 1$, and for $n \geq 1$,

$$c_n = n \cdot c_{\lfloor n/2 \rfloor}.$$

Use mathematical induction to prove that $c_n < n!$ for $n \geq 3$.