Midterm 1 Fall 2016

October 14th, 2016

2	10	1
3	10	9.
4	10	10.
5	10	∅
6	5	4
Total:	50	28

Instructions:

- You will have 50 minutes for the exam.
- There are 6 questions.
- All proofs should be in complete sentences.
- No notes, books, iPhones, or electronic devices may be used during the exam.
- To obtain full credit, you must clearly show all of your work except for the Multiple Choice and True/False questions.
- Good luck!

1. (5 points) Circle the *one* correct answer.

   (a) (1 point) Consider the relation on $\mathbb{N}$ determined by divisibility. That is to say, $a \sim b$ whenever a divides b. This is a:

   A. total order.
   B. symmetric relation.
   C. partial order.
   D. function.
   E. A and C.

   (b) (1 point) Let $f : \mathbb{R} \to \mathbb{R}$ be given by the rule $f(x) = x^n$ where $n \in \mathbb{N}$. For what values of n is the function f bijective?

   A. $n = 0, 1, 3$
   B. n odd.
   C. n even.
   D. For no such $n \in \mathbb{N}$.
   E. All $n \in \mathbb{N}$.

   (c) (1 point) How many functions are there from $\{1, 2, 3, 4, 5\}$ to $\{6, 7\}$?

   A. None
   B. 10
   C. 20
   D. 25
   E. 32

   2^5

   (d) (1 point) How many reflexive relations are there on $S \times S$ where $S = \{1, 2\}$?

   A. 1
   B. 2
   C. 3
   D. 4
   E. 8

   $\{1,2\}$

4

2. (10 points) (a) (3 points) Carefully define what a partial order is.

$\emptyset$

(b) (3 points) Let X be a set and $\mathcal{P}(X)$ be its power set Carefully show that $\subseteq$ defines a partial order on $\mathcal{P}(X)$. You may use the following set-theoretic facts without proof:

PARTIAL ORDER

S1. $A \subseteq B \iff$ for every $x \in A$, $x \in B$.

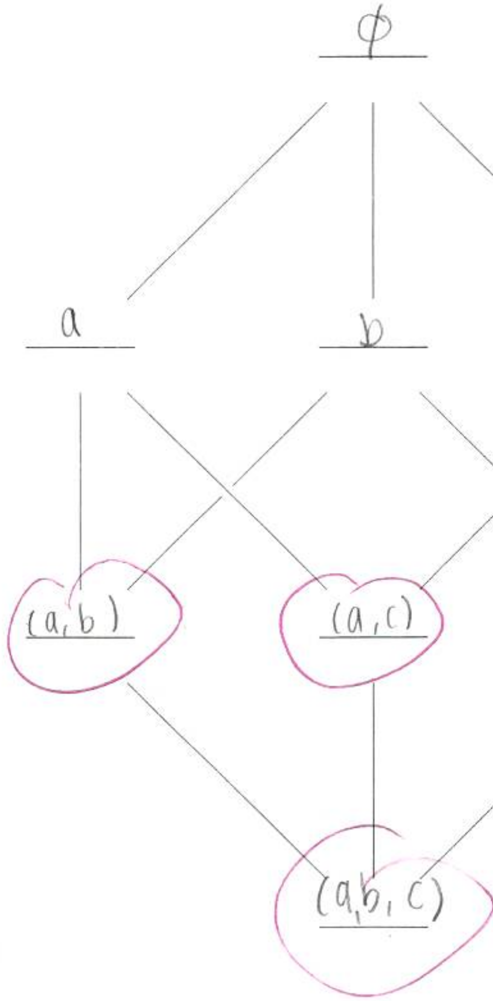
S2. $A \subseteq B$ and $B \subseteq A \iff A = B$.

X is a subset of P(X) because P(X) contains all the subsets of X, including X

↳ S1 proof

$\emptyset$

(c) (4 points) Fill in the Hasse Diagram below for the partial order $(X, \subseteq)$ where $X = \{a, b, c\}$. Please draw an arrow head from A to B if $A \subseteq B$.



?
$\{a, b, c\}$

+1

3. (10 points) Use induction to prove for all $n \geq 1$:

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}.$$

base case: $n=1$

$$\frac{1}{(2)(1)+1} = \frac{1}{3} = \frac{1}{1\cdot 3} \quad \checkmark$$

inductive step: assume $n=k$

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$$

prove true for $n+1$

$$\underbrace{\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2k-1)(2k+1)}}_{\frac{k}{2k+1}}+\frac{1}{(2k+1)(2k+3)} = \frac{k+1}{2(k+1)+1}$$

$$\left[\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)} = \frac{k+1}{2k+3}\right](2k+3)(2k+1)$$

$$(k)(2k+3)+(1) = (k+1)(2k+1)$$

$$2k^2+3k+1 = 2k^2+2k+1 \quad \checkmark$$

-1

don't ~~assume~~ assume LHS = RHS.

+9

4. (10 points) There was a survey given to vegans on their dietary choice. It had the following options that could be checked:

   - ☐ I am a vegan for enviornmental reasons. 22
   - ☐ I am a vegan to protect the lives of animals. 25
   - ☐ I am a vegan for personal health. 39

   86

   Each person surveyed could check any number of boxes that applied. Some did not check any at all. It was found that:

   - 22 are vegans for environmental reasons.
   - 25 are vegans to protect the lives of animals.
   - 39 are vegans for personal health.
   - 9 are vegans for environmental reasons and to protect the lives of animals.
   - 17 are vegans for environmental reasons and for personal health.
   - 20 are vegans to protect the lives of animals and for personal health.
   - 6 are vegans for all the provided reasons.
   - 4 did not provide any reason for their dietary choice.

   How many vegans were surveyed?
   Note: Those 17 that stated they are vegans for environmental reasons and to protect the lives of animals are included in the group of 22 that stated they were vegans for environmental reasons. This similarly applies to all the "and" statements above.

$$|A\cup B\cup C| = |A| + |B| + |C| - |(A\cap B) + (A\cap C) + (B\cap C)| + |A\cap B\cap C|$$

$$= 22 + 25 + 39 - |9 + 17 + 20| + 6 + 4$$

$$= 86 - 46 + 10 = 56$$

$$\begin{array}{r} 22 \\ 25 \\ 39 \\ \hline 86 \end{array} \qquad \begin{array}{r} 29 \\ 17 \\ \hline 46 \end{array}$$

what is A, B, C, .. +0

5. (10 points) Show by contradiction there is no surjection from $f : \mathbb{N} \to (0,1)$.
Hint 1: This is similar to the homework problem that there is no surjection from $f : X \to \mathcal{P}(X)$.
Hint 2: Note every number in (0, 1) has an infinite decimal expansion $.r_1r_2r_3r_4\ldots$ where $r_i \in \{0, 1, \ldots, 9\}$. Take any f and consider writing the rule as follows:

$$\begin{aligned}
1 &\mapsto .r_1^1r_2^1r_3^1r_4^1\ldots \\
2 &\mapsto .r_1^2r_2^2r_3^2r_4^2\ldots \\
3 &\mapsto .r_1^3r_2^3r_3^3r_4^3\ldots \\
4 &\mapsto .r_1^4r_2^4r_3^4r_4^4\ldots \\
&\vdots
\end{aligned}$$

$(0,1) = \{x \mid 0 < x < 1\}$

Now, find a number $y \in (0,1)$ *not* in the codomain. 0

to show by contradiction,

$\emptyset$

6. (5 points) Determine whether each statement is true or false and determine the correct answer.

(a) True False Let f and g be functions. If $f \circ g$ is surjective, then g is surjective.

(b) True False Let f and g be functions. If f and g are surjective, then $f \circ g$ is surjective.

(c) True False All total orders are partial orders.

(d) True False The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = e^x$ is surjective.

(e) True False $8^{20} - 1$ is divisible by 7.