Student name:

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MATH 61 (Butler)
Midterm I, 20 October 2008

This test is closed book and closed notes, with the exception that you are allowed one $8\frac{1}{2}'' \times 11''$ page of handwritten notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. You do not need to simplify terms such as "5!" or "$\binom{13}{5}$" (we would prefer if you didn't even try!). Each problem is worth 10 points.

1. Stan's Donuts is considering selling donuts in packs of three or seven (the "Baker's half-dozen"). Show, using induction, that if Stan's makes this change that for any number of donuts $k \geq 12$ it will always be possible to purchase k donuts.

$k=12$ - 4 packs of three
$k=13$ - 2 packs of three, 1 pack of 7
$k=14$ - 2 packs of 7
$k=15$ - 5 packs of three
$k=16$ - 3 packs of three, 1 pack of 7

We can go from $k+3$ to k donuts by removing a single 3 pack (or k to $k+3$ by adding a 3 pack). So since we know we can get $k=12, 13, 14$, we will be able to get to any k. I say this because $12+3n$, $13+3n$, $14+3n$ will cover all integers $k \geq 12$.

How are you doing induction?

-3

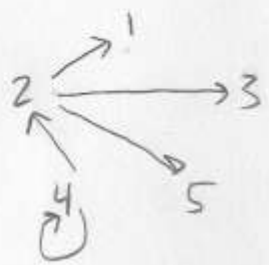
1	7
2	10
3	10
4	10
5	10
Σ	47

2. Let $X = \{1, 2, 3, 4, 5\}$ and form the relationship R by aRb if $a + 2b$ is a multiple of 4.

odd plus even = odd

(a) We have looked at three representations of relations; as a *set*, as a *graph*, and as a *matrix*. Produce two of the three representations for the relation R.

graph



matrix

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b) Which of the following properties does this relation have: *reflexive*, *symmetric*, *anti-symmetric*, *transitive*, *partial order* and *equivalence*. (You do not need to justify your answer.)

anti-symmetric

No other properties apply.

3. For this problem we will consider the word "BOOKKEEPER".

(a) How many ways are there to rearrange the letters of the word BOOKKEEPER?

$$\frac{10!}{1!2!2!3!1!1!} = \boxed{\frac{10!}{2!2!3!}}$$

(b) How many ways are there to rearrange the letters of the word BOOKKEEPER if no two E's can be adjacent. (For example EROOKKEBEP has no two adjacent E's so hould be counted but BEEPROKOKE has two adjacent E's so should not be counted.)

Take out Es

_B_O_O_K_K_P_R_

can arrange in $\frac{7!}{2!2!}$

choose 3 slots from 8 possible for E_s: $\binom{8}{3}$

so we have: $\boxed{\frac{7!}{2!2!}\binom{8}{3}}$

4. Let $X = \{1,2,3,4,5\}$, let $f : X \to X$ by $f(1) = 3, f(2) = 2, f(3) = 5, f(4) = 1, f(5) = 4$ and let $g : X \to X$ by $g(1) = 4, g(2) = 1, g(3) = 2, g(4) = 5, g(5) = 3$.

Is there a function $h : X \to X$ so that $h \circ f = g$ (i.e., so that for all x that $(h \circ f)(x) = g(x)$)? If there is a function, write one down. If not, prove there cannot be one.

f:

g:

$h(f(1)) = h(3) = g(1)$
$h(f(2)) = h(2) = g(2)$
$h(f(3)) = h(5) = g(3)$
$h(f(4)) = h(1) = g(4)$
$h(f(5)) = h(4) = g(5)$

Yes there is. Here it is:

$$\boxed{\begin{aligned} h(1) &= 5 \\ h(2) &= 1 \\ h(3) &= 4 \\ h(4) &= 3 \\ h(5) &= 2 \end{aligned}}$$

5. The local Pizza Shack currently has a coupon where you can get either a two topping pizza plus a soda OR a three topping pizza without a soda for fifteen bucks. If there are 10 different types of toppings and 6 different types of soda how many different meal possibilities can you choose from if you use the coupon?
(Note: all the toppings on the pizza must be different.)

option 1 : 3 toppings

$$\binom{10}{3}$$

option 2 : 2 toppings + soda

$$\binom{10}{2}\binom{6}{1}$$

$$\boxed{\binom{10}{3} + \binom{10}{2}\binom{6}{1}}$$