

Student name:

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MATH 61 (Butler)
Midterm I, 20 October 2008

This test is closed book and closed notes, with the exception that you are allowed one $8\frac{1}{2} \times 11$ " page of handwritten notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. You do not need to simplify terms such as "5!" or " $\binom{13}{5}$ " (we would prefer if you didn't even try!). Each problem is worth 10 points.

1. Stan's Donuts is considering selling donuts in packs of three or seven (the "Baker's half-dozen"). Show, using induction, that if Stan's makes this change that for any number of donuts $k \geq 12$ it will always be possible to purchase k donuts.

donuts

base cases:

$k = 12:$	$4 \cdot 3 + 0 \cdot 7 = 12$
$= 13:$	$2 \cdot 3 + 1 \cdot 7 = 13$
$= 14:$	$0 \cdot 3 + 2 \cdot 7 = 14$

assuming it is possible to purchase k donuts $15 \leq k \leq n$,
to purchase $(n-3)$ donuts just purchase one less pack of
3 donuts. ?? Aren't we trying to buy more donuts?

-3

1	7
2	8
3	10
4	7
5	10
Σ	42

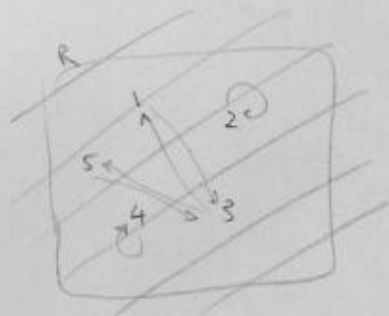
2. Let $X = \{1, 2, 3, 4, 5\}$ and form the relationship R by aRb if $a + 2b$ is a multiple of 4.

-2 (a) We have looked at three representations of relations; as a *set*, as a *graph*, and as a *matrix*. Produce two of the three representations for the relation R .

$$R = \{(1, 3), (2, 2), (3, 1), (3, 5), (4, 4), (5, 3)\}$$

~~(a)~~ You want $a + 2b$ not $a + b$ divisible by 4

$$A_R = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}$$



~~(b)~~ (b) Which of the following properties does this relation have: *reflexive*, *symmetric*, *anti-symmetric*, *transitive*, *partial order* and *equivalence*. (You do not need to justify your answer.)

the relation is ~~symmetric~~.

~~anti-symmetric~~ (ok, since you did it right for your wrong a)

$$A_R^2 = \begin{pmatrix} 1 \\ \vdots \end{pmatrix}$$

(not transitive)

3. For this problem we will consider the word "BOOKKEEPER".

(a) How many ways are there to rearrange the letters of the word BOOKKEEPER?

1 B, 2 O, 2 K, 3 E, 1 P, 1 R and

10 letters:
$$\frac{10!}{1! 2! 2! 3! 1! 1!}$$

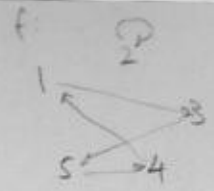
(b) How many ways are there to rearrange the letters of the word BOOKKEEPER if no two E's can be adjacent. (For example EROOKKEBEP has no two adjacent E's so should be counted but BEEPROKOKKE has two adjacent E's so should not be counted.)

B O O K K P R

place 7 non-E letters.
$$\frac{7!}{1! 2! 2! 1! 1!}$$
 ways

fill 3 E's into gaps (8 to choose from) $\binom{8}{3}$ ways

by multiplication principle:
$$\frac{7!}{2! 2! 1! 1! 1!} \binom{8}{3}$$



4. Let $X = \{1, 2, 3, 4, 5\}$, let $f : X \rightarrow X$ by $f(1) = 3, f(2) = 2, f(3) = 5, f(4) = 1, f(5) = 4$ and let $g : X \rightarrow X$ by $g(1) = 4, g(2) = 1, g(3) = 2, g(4) = 5, g(5) = 3$.

Is there a function $h : X \rightarrow X$ so that $h \circ f = g$ (i.e., so that for all x that $h(f(x)) = (h \circ f)(x) = g(x)$)? If there is a function, write one down. If not, prove there cannot be one.

$h(f(x)) = (h \circ f)(x) = g(x)$

- $f(1) = 3 \quad h(3) = 4$
- $f(2) = 2 \quad h(2) = 1$
- $f(3) = 5 \quad h(5) = 2$
- $f(4) = 1 \quad h(1) = 5$
- $f(5) = 4 \quad h(4) = 3$

$h : X \rightarrow X$
 $\forall x \in X, (h \circ f)(x) = g(x)$

5. The local Pizza Shack currently has a coupon where you can get either a two topping pizza plus a soda OR a three topping pizza without a soda for fifteen bucks. ???!
If there are 10 different types of toppings and 6 different types of soda how many different meal possibilities can you choose from if you use the coupon?
(Note: all the toppings on the pizza must be different.)

case 2 toppings + 1 soda:

$$\binom{10}{2} \cdot \binom{6}{1}$$

case 3 toppings, no soda:

$$\binom{10}{3}$$

by addition principle: number of different meals to choose from is

$$\boxed{\binom{10}{2} \cdot \binom{6}{1} + \binom{10}{3}}$$