Math 61, Lec 1
Winter 2016
Exam 1
1/25/16
Time Limit: 50 Minutes

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Discussion Section: 1A

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing.

You may *not* use books, notes, or any calculator on this exam.

If your answer contains a number that is impossible to simplify without the use of a calculator, such as e^3, $\ln(3)$ or $\sin(3)$, you may leave answers in terms of e, ln, or trig functions.

Partial credit will only be awarded to answers for which an explanation and/or work is shown.

Please attempt to organize your work in a reasonably neat and coherent way, in the space provided. If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	7	7
2	18	5
3	25	22
4	25	25
5	25	12
Total:	100	71

1. (7 points) Negate the following implication:
"If you are on the wait list, then you will be enrolled in the class."

p = you are on the wait list

q = you will be enrolled in the class

negate $p \to q$

$\neg(p \to q) = p \wedge \neg q$

So the negation is

If you are on the waitlist and you will not be enrolled in the class

2. (18 points) (a) Let X be a set with n elements. How many different relations on X are there?

There will be $n \times n = n^2$ different relations on X.

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(b) Let X be a set with n elements. How many different relations on X are there that are not reflexive?

there are n different relations on X that are reflexive.

So, there will be $n^2 - n$ different relation on X that are not reflexive

$2^{n^2} - 2$

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(c) Let X be a set with n elements and let Y be a set with m elements, where $m \geq n$. How many different one-to-one functions from X to Y are there?

one-to-one is if every $x \in X$ maps to $y \in Y$ that are unique to each other such that $f(x) = y$.

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So in this case there will be n different one-to-one functions from X to Y.

3. (25 points) Use mathematical induction to prove that $1+3+5+\cdots+(2n+1)=(n+1)^2$ for every integer $n \geq 0$.

Base case: let $n = 0$

$2(0)+1 = (0+1)^2$

$1 = 1$ QED

Induction step: Assume $1+\cdots+2n+1 = (n+1)^2$ for some $n \geq 0$ −3

prove that : $1+3+5+\ldots+(2n+1)+(2(n+1)+1) = ((n+1)+1)^2$.

add $(2(n+1)+1)$ to both side. we get:

$1+3+5+\ldots+(2n+1)+(2(n+1)+1) = (n+1)^2+(2(n+1)+1)$

RHS: $(n+1)^2+(2(n+1)+1)$

$= n^2+2n+1+2n+2+1$

$= n^2+4n+4$

$= (n+2)^2$

$= ((n+1)+1)^2$ QED

4. (25 points) Let X be a set. We define the *power set* $\mathcal{P}(X)$ to be the set of all subsets of X. For example, if $X = \{1, 2\}$, then $\mathcal{P}(X) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.

Define a relation R on $\mathcal{P}(\mathbb{Z})$ by $(S, T) \in R$ if and only if $S \subseteq T$, for any sets S and T in $\mathcal{P}(\mathbb{Z})$. Prove that R is a partial order.

R is a partial order if R is reflexive, anti-symmetric, and transitive

✓ prove reflexive: prove $(S,S) \in R$

$S \subseteq S$ is true, so R is reflexive

prove anti symmetric: prove $(S,T) \in R$ and $(T,S) \in R \Rightarrow S = R$.

If $S \subseteq T$, then for every $s \in S$, there exist $s \in T$, so $\forall s \in S \cap T$

✓ and if $T \subseteq S$, then for every $t \in T$, there exist $t \in S$, so $\forall t \in S \cap T$

This condition could only happen if $S = T$ so R is antisymmetric

prove transitive: prove $(S,T) \in R$ and $(T,U) \in R \Rightarrow (S,U) \in R$.

If $S \subseteq T$, then for every $s \in S$, there exist $s \in T$

✓ If $T \subseteq U$, then for every $t \in T$, there exist $t \in U$.

This condition proves that for every $s \in S$, there exist $s \in U$

So $(S,U) \in R$, thus R is transitive

Since R is reflexive, anti symmetric, and transitive,

R is partial order.

5. (25 points) As in the previous problem, for any set X, we define the *power set* $\mathcal{P}(X)$ to be the set of all subsets of X.
Consider the sets $X = \{1, 2, 3\}$ and $Y = \{4, 5, 6\}$.
Define a function $f : \mathcal{P}(X) \times \mathcal{P}(Y) \to \mathcal{P}(X \cup Y)$ by $f((S,T)) = S \cup T$, for $S \in \mathcal{P}(X)$ and $T \in \mathcal{P}(Y)$. Prove that f is a bijection.

f is a bijection if it is one-to-one and onto.

prove one-to-one:

$f((S_1, T_1)) = f((S_2, T_2))$

$S_1 \cup T_1 = S_2 \cup T_2$

$S_1 + T_1 - \cancel{S_1 \cap T_1} = S_2 + T_2 - \cancel{S_2 \cap T_2}$ → since P(x) and P(Y) are mutually exclusive

NO

$S_1 + T_1 = S_2 + T_2$

so $S_1 = S_2$ and $T_1 = T_2$.

WHY

QED

prove onto:

let $\beta \in S \cup T$,

?

prove that for any value of β there exist (S,T) such that

$f((S,T)) = \beta$

NOPE

$f((S,T)) = S \cup T$

Since $S \cup T$ means to combine all sets in S and T, then $S \in S \cup T$ and $T \in S \cup T$

so there will always exist (S,T) such that $f((S,T)) = S \cup T$.

QED

Since f is one-to-one and onto, f is bijective

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