Math 61-1 Midterm 2 version a

BENJAMIN HE

TOTAL POINTS

37.5 / 50

QUESTION 1

Multiple choice 10 pts

1.1 **2 / 2**

✓ **- 0 pts** Correct (d)

- 2 pts Incorrect

1.2 **2 / 2**

✓ **- 0 pts** Correct (a)

- 2 pts Incorrect

1.3 **2 / 2**

✓ **- 0 pts** Correct (c)

- 2 pts Incorrect

- 2 pts No Answer

1.4 **2 / 2**

✓ **- 0 pts** Correct (b)

- 2 pts Incorrect

1.5 **2 / 2**

✓ **- 0 pts** Correct (d)

- 2 pts Incorrect

- 2 pts No Answer

QUESTION 2

Short answer 10 pts

2.1 **2 / 2**

✓ **- 0 pts** Correct (176)

- 2 pts Incorrect

- 2 pts No answer

2.2 **2 / 2**

✓ **- 0 pts** Correct (34650)

- 2 pts Incorrect

2.3 **2 / 2**

✓ **- 0 pts** Correct (21)

- 2 pts Incorrect

- 2 pts No answer

2.4 **0.5 / 2**

- 0 pts Correct (960)

- 2 pts Incorrect

- 0.5 pts Gave term, not just coefficient

✓ **- 1.5 pts** Didn't multiply by 8 (120) or similar

- 2 pts No answer

2.5 **0 / 2**

- 0 pts Correct (302400)

✓ **- 2 pts** Incorrect

- 2 pts No answer

QUESTION 3

Euler paths 10 pts

3.1 criteria for euler path **2 / 5**

+ 5 pts Correct

✓ **+ 2 pts** Euler cycle criterion

+ 1 pts Euler cycle criterion (missing connected, or other mistake)

+ 2 pts Reduction to graph with even degrees

+ 1 pts Reduction to graph with even degrees (with mistake)

+ 1 pts Correct explanation of how to get Euler path from Euler cycle

+ 0 pts Incorrect

3.2 application of euler paths **6 / 5**

✓ **- 0 pts** Correct

✓ **+ 1 pts** Click here to replace this description.

+ 2 pts Click here to replace this description.

QUESTION 4

4 Pigeon hole **10 / 10**

✓ - **0 pts** **Correct**

- **3 pts** Incorrect Partition
- **3 pts** No Pigeonhole
- **2 pts** Minor error
- **10 pts** Blank

QUESTION 5

Hypercube 10 pts

5.1 recurrence for edges **1 / 4**

- **0 pts** Correct
- **4 pts** empty
- **2 pts** large mistake
- **2 pts** you are assuming the desired conclusion
- **1 pts** need to explain how the hypercube it built out of smaller ones
- **3 pts** can't just do examples

✓ - **1 pts** **incomplete**

- **3 pts** I don't see how this shows that the recurrence is true

- 2 Point adjustment

right idea

5.2 number of edges **2 / 6**

- **0 pts** Correct
- **1 pts** Need to use induction to show that formula is true

✓ - **4 pts** **wrong answer, need to use iteration**

- **6 pts** empty
- **3 pts** wrong answer, this is why you need to use induction to show that answer is true
- **4 pts** incomplete
- **2 pts** wrong answer

gradescope

J. Cameron Math 61 Mon, February 25th, 2019

Midterm 2

Name: Benjamin He

Student ID: 804962948

Section:

	Tuesday:	Thursday:	
	1A	1B	TA: Albert Zheng
	1C	1D	TA: Benjamin Spitz
	1E	1F	TA: Eilon Reisin-Tzur

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators,** books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper, nor may you work on the exam after it has ended. There is some scratch paper at the back of the exam, if you do work there indicate so on the relevant question. Do not write on the backs of the pages. Please circle or box your final answers.

Please get out your id and be ready to show it when you turn in your exam.

Please do not write below this line.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Circle the correct answer (only one answer is correct for each question)

1. K_n has an Euler cycle:
   (a) For every n
   (b) For no n
   (c) For n even
   (d) For n odd

2. If X is a set with 10 elements and Y is a set with 3 elements and $f : X \to Y$:

   $\lceil \frac{10}{3} \rceil = 4$

   (a) there are at least 4 distinct elements of X, x_1, x_2, x_3, x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4)$.
   (b) it is possible that there are *not* 4 distinct elements of X, x_1, x_2, x_3, x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4)$.
   (c) there are at least 5 distinct elements of X, x_1, x_2, x_3, x_4, x_5 with $f(x_1) = f(x_2) = f(x_3) = f(x_4) = f(x_5)$.
   (d) f could be one-to-one

3. If $s_0 = 2$ and $s_1 = 1$ and for $n \geq 2$ $s_n = s_{n-1} + s_{n-2}$ then $s_n =$
   (a) $\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$
   (b) $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$
   (c) $\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$
   (d) none of the above

Question 1 continues on the next page...

4. The number of ways of arranging a math book, a computer science book, and an English book on a shelf is: + history
   (a) 4
   (b) 4!
   (c) $1 + 2 + 3 + 4$
   (d) $4 - 3 + 2 - 1$

5. The number of relations on a set with n elements that are both symmetric and reflexive is:
   (a) 2^{n^2}
   (b) $2^{n^2-n} + 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$
   (c) $2^{n^2} + 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$
   (d) none of the above

$2^{\frac{n^2-n}{2}}$

$9 - 3 = 3$

$\{1, 2, 3\}$

1,2
2,1

1,1	2,1	3,1
1,2	2,2	3,2
1,3	2,3	3,3

2^4

2. In this question write down your answer, no need for any justification. Leave your answers in a form involving factorials, $P(n, m)$, $\binom{n}{m}$, exponents, etc.

(a) (2 points) What is the number of solutions to the equation $x_1 + x_2 + x_3 = 20$ where $x_3 \leq 10$ and x_1, x_2, x_3 are nonnegative integers?

$\binom{22}{2} - \binom{11}{2}$ → Total − (# if $x_3 > 10$)

(b) (2 points) What is the number of ways of rearranging the letters of the word "MISSISSIPPI"?

$\frac{11!}{4!\,4!\,2!}$

(c) (2 points) What is the number of binary strings of length 6 that don't contain 11 as a substring?

$10 \underline{\quad}$ b_{n-2}

$0 \underline{\quad}$ b_{n-1}

$b_n = b_{n-1} + b_{n-2}$

$a_0 = 1$ $a_4 = 8$
$a_1 = 2$
$a_2 = 3$
$a_3 = 5$

$a_n = a_{n-1} + a_{n-2}$

$a_6 = a_5 + a_4 = a_4 + a_3 + a_3 + a_2 = 8 + 2(5) + 3$

$= \boxed{21}$

(d) (2 points) What is the coefficient of x^7y^3 in the polynomial $(x+2y)^{10}$?

$\binom{10}{7}\binom{3}{3}$

(e) (2 points) I have 10 distinct books. Two are by Ernest Hemingway and three are by Virginia Woolf. How many ways are there to arrange these books on a shelf if the ones by Hemingway must be next to each other and the ones by Woolf must not be next to each other?

$2(9!) - 2(8!)(3!)$

Total when hemingway are together

total when hem & woolf are together

3. (a) (5 points) Show that if a connected graph has exactly 2 vertices of odd degree, then there is a path in the graph that visits each edge exactly once. (Feel free to use what we know about Euler cycles from lecture)

We know that for an Euler cycle to exist, the degrees of all vertices must be even. However, that's because the Euler cycle returns to its original vertex. For a path that ~~doesn't~~ isn't required to be a cycle, we can simply remove the edge that connects the start vertex of the path to the end vertex of the path, thus making exactly those two ~~vertex~~ vertices' degrees odd while the rest remain even. Because of this, you would also know that the path given in the problem would have to begin from one of the odd-degree vertices and end at the other odd-degree vertex. (This is all assuming the graph is connected as well).

Question 3 continues on the next page...

(b) (5 points) A house has a front door and a back door and no other doors leading outside. Inside each room of the house, there are (exactly) two doors that each go to other rooms of the house [1] (and the only way to enter or leave a room is through a door). Each room can be reached from the front door. Show that it is possible to go in through the front door and leave through the back door while going through each door in the house exactly once. Feel free to use the previous part of this problem.

Only the front and back doors have 3 doorways or "edges" meaning they have deg 3 which is odd. The rest of the doorways have degree 2 which are even. ~~Thus~~ We also know the doorways are "connected" b/c they can all be reached from the front & back door due to the footnote. Thus, using the last problem's conclusion, we can say that there is a "Euler" path going from the front door through the back that passes through every door exactly once.

[1] To clarify the rooms just inside the front and the back doors have three doors, one of them is the front/back door, and the other two are the doorways leading to other rooms of the house

4. (10 points) Suppose that six distinct integers are selected from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Prove that at least two of these six integers sum to 11.

Suppose we have a set of 6 distinct numbers (the ones selected from the set)

$S_1 = \{a_1, a_2, a_3 \dots a_6\}$

and another set

$S_2 = \{11-a_1, 11-a_2, 11-a_3, \dots 11-a_6\}$

The maximum range of any of these numbers is from 1 to 10, however there are 12 total elements in the combined set of $S_1 \cup S_2$.

By pidgeonhole principle at least two of these 12 elements are the same value. However because we know all the elements in S_1 are distinct from each other and all the elements of S_2 are distinct from each other, that must mean that one element in S_1 is identical to one element in S_2. Thus, we know that there must be some 2 elements in S_1 where

$a_i + a_j = 11$ and $i \neq j, a_i, a_j \in S_1$. Because S_1 represents the 6 distinct integers chosen in the problem, we can conclude that at least 2 of the 6 integers sum to 11.

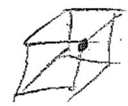
5. Recall the n-dimensional hypercube. This is a graph with vertices labeled by binary strings of length n, with an edge between two vertices if they differ in exactly one digit. Let c_n be the number of edges in the n-dimensional hypercube.

(a) (4 points) Show that c_n satisfies the recurrence $c_n = 2^{n-1} + 2c_{n-1}$

2^n vertizes

degree of every vertize in n-d = $n-1$

$2^n(n-1)$



12 $C_3 = 12$

$C_2 = 4$

$C_1 = 1$

$C_0 = 0$

↑

every additional dimension doubles the # vertices which can be represented in the recurrence.

Question 5 continues on the next page...

Question 5 continued...

(b) (6 points) Solve your recurrence from the previous part of this question to find a formula for c_n.

$$c_n = 2^{n-1} + 2c_{n-1}$$

Assume $c_n = t^n$ $\rightarrow$ $t^n = 2^{n-1} + 2t^{n-1} = 2(2^{n-2} + t^{n-1})$

$$t^n - 2t^{n-1} - 2^{n-1} = 0$$

$$t^{n-1}(t-2) - 2^{n-1} = 0$$

$$t^{n-1}(t-2) = 2^{n-1}$$

$$\frac{1}{2}t^n = 2^{n-2} + t^{n-1}$$

$$\cancel{\frac{1}{2}t^n - t^{n-1} = 2^{n-2}}$$

$$\boxed{t_n = 2^n(n-1)}$$

$$t_{n-1} = 2^{n-1}(n-2)$$

$$2^{n-1} + 2^n(n-2)$$

This page has been left intentionally blank. You may use it as scratch paper. It will not be graded unless indicated very clearly here and next to the relevant question.