Midterm I(A): The First

N

January 30 2017

This test totals 45 points and you get 50 minutes to do it. Answer the questions in the spaces provided on the question sheets. Show work unless the question says otherwise. Good luck!

Name : Simeng Pang

ID number : 604625053 Section : 1E

Question	Points	Score
1	10	10
2	10	10
3	10	10
4	5	5
5	10	8
Total:	45	43

1. (10 points) Using induction, show that 6 divides $n^3 - n$ for all integers $n \geq 1$.

Base case : $P(1)$: $1^3 - 1 = 0$

0 divids 6 , it's true for $P(1)$

Induction hypothesis

If we assume $P(N)$ is true , which means

6 divides $n^3 - n$, $n^3 - n = 6a$

we want to show $P(N+1)$ is true

$P(N+1)$: $(n+1)^3 - (n+1)$

$= (n^2+2n+1)(n+1) - (n+1)$

$= (n^2+2n+1-1)(n+1)$

$= (n^2+2n)(n+1)$

$= n^3 + 3n^2 + 2n$

since $n^3 - n = 6a$ $n^3 = 6a + n$

$= 6a + 3n + 3n^2$

$= 6a + 3n(n+1)$

because n, $n+1$ are consecutive, there must be an even number in n, $n+1$

So $3n(n+1)$ is even

$= 6a + 6\frac{n(n+1)}{2}$

$= 6\left(a + \frac{n(n+1)}{2}\right)$ where $a + \frac{n(n+1)}{2}$ is integer

so $P(N+1)$ is true.

From induction, we know that 6 divides $n^3 - n$

. A survey was conducted among 24 people about three ice-cream flavours (chocolate, butterscotch and strawberry). It was found that the number of people who liked chocolate was the same as the number of people who liked butterscotch which was also the same as the number of people who liked strawberry. It was also found that 7 people liked none of these flavours, 5 people liked all these flavours, 7 people liked both butterscotch and chocolate, 6 people liked both chocolate and strawberry and 5 people liked both butterscotch and strawberry. Let U represent the set of 24 people surveyed, C represent the set of people who liked chocolate, S the set of people who liked strawberry and B, the set of people who liked butterscotch. Answer the following questions:

No need to show work for this question. However a Venn diagram is given in case you'd like to use it. Also no partial credit will be given for this question. So double check your answers!

(a) (2 points) What is $|\overline{C \cup B \cup S}|$?

7

(b) (2 points) What is $|C \cap B \cap S|$?

5

(c) (2 points) What is $|S \cap B|$?

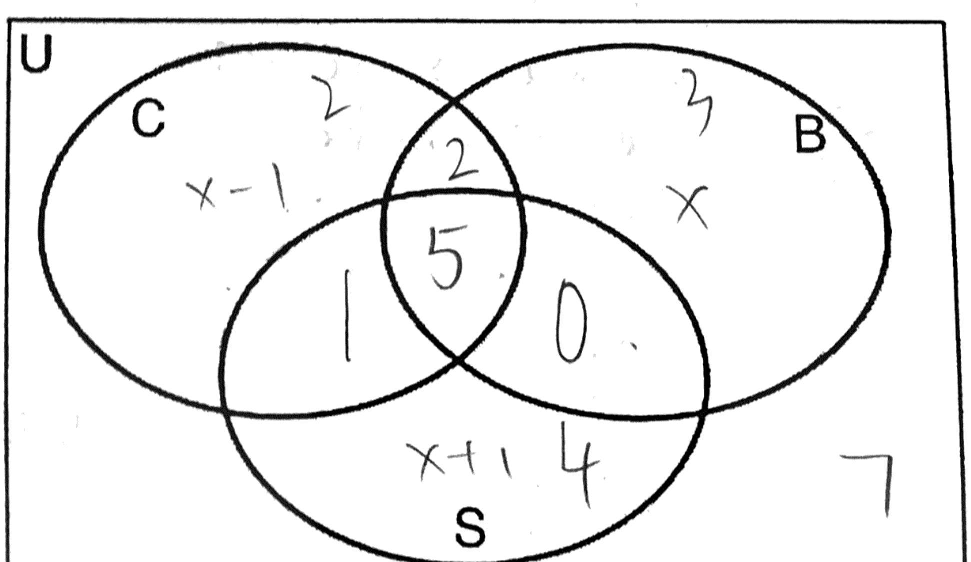
5

(d) (2 points) How many people liked chocolate flavour?

$2+2+5+1 = 10$ people

(e) (2 points) How many people liked chocolate flavour but did not like strawberry nor butterscotch ?

2 people



$3x+8=17.$
$x=3$

3. Consider the following relation T defined on $\mathbb{R}$, the set of real numbers.

$$(x, y) \in T \text{ if and only if } x - y \leq 1.$$

Answer the following questions, FULLY JUSTIFYING YOUR ANSWER. If your answer is yes, explain why. If your answer is no, give a counter-example.

(a) (2 points) Is T a function ?

No. Because if $x=1$ y can be any number bigger or equal to 0, but for function $x \in X$ can only have one output $y \in Y$, so it's not a function.

2

(b) (2 points) Is T reflexive ?

Yes. Because if we pick $a \in \mathbb{R}$
$a - a = 0 \leq 1$ so (a,a) is in
so T is reflexive.

2

(c) (2 points) Is T symmetric ?

No. Take $(3,5)$ and $(5,3)$
$3-5 = -2 \leq 1$ $5-3 = 2 > 1$ so $(5,3)$ is not in T.
so T is not symmetric.

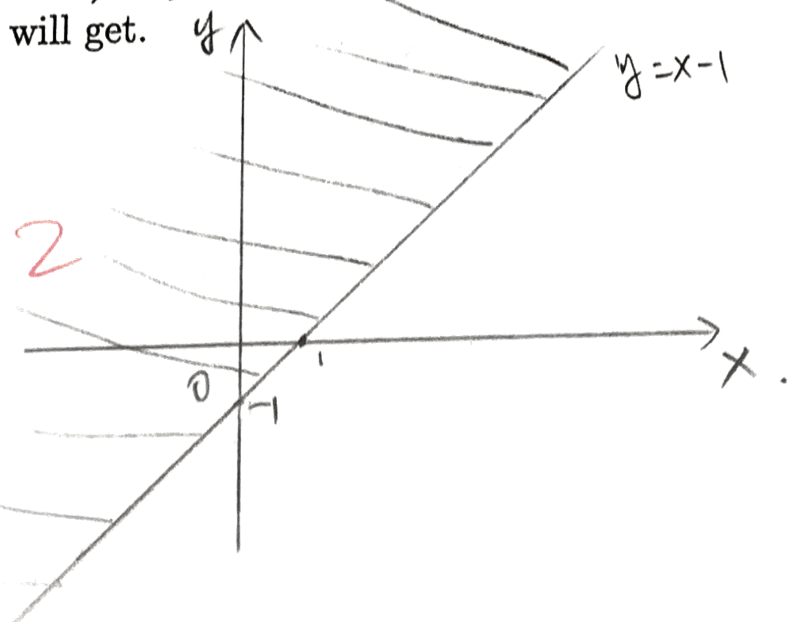
2

(d) (2 points) Is T transitive ?

No. Take $(4,3)$, $(3,2)$
$4-3 = 1 \leq 1$ $3-2 = 1 \leq 1$ but $4-2 = 2 > 1$
so $(4,2)$ is not in T
so T is not transitive.

2

(e) (2 points) If you plot points of T on the real plane, draw a rough graph of what you will get.



2

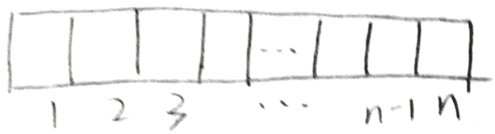
$x - y \leq 1$

$y \geq x - 1$

we will get the shaded area and the line $y = x-1$

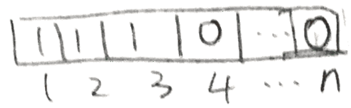
4. (5 points) For an integer $n \geq 1$, let $Y = \{1, 2, 3, \ldots, n\}$ and let $P(Y)$ denote the power set of Y. Let $X = \{0, 1\}$ and let Z denote the set of all strings on X of length n. Construct a bijection $f : P(Y) \to Z$ (Explain carefully what the function is and why it is a bijection)

$P(Y) \to Z$



The function is for each set $a \in P(Y)$, we construct a n-bit string that represents the presence of each element in a. If any number from a ~~to~~ n is not in a, the corresponding bit of string will be 0, otherwise the corresponding bit of string is 1.

For example

$\{1, 2, 3\} \longrightarrow$ 

It's injective because if string a = string b, 1s and 0s are exactly same, what they represent are also same.

so set a = set b

It's surjective because from any $s \in Z$, we can find corresponding set $a \in P(Z)$ according to 1s and 0s

5. (10 points) Circle the correct answer. No need to show work for this question. (Also no partial credit)

1a 2a 1a 2b 1a 2c
2a 1b

(a) Number of functions from $\{1, 2\}$ to $\{a, b, c\}$

A. 8 B. 9 C. 6 D. ∞ E. None of these.

1 2 a b c

(b) Number of onto functions from $\{1, 2\}$ to $\{a, b, c\}$

A. 6 B. 5 C. 0 D. 2 E. None of these.

1 2 a b c

(c) Number of one-one functions from $\{1, 2\}$ to $\{a, b, c\}$

A. 6 B. 5 C. 0 D. 2 E. None of these.

8

(d) Number of relations from $\{1, 2\}$ to $\{a, b, c\}$

A. 6 B. 2^5 C. ∞ D. 2^6 E. None of these.

1 2 a b c

(e) Number of equivalence relations on $\{1, 2, 3\}$

A. 7 B. 5 C. 8 D. ∞ E. None of these.

(1,1) (1,2) (1,3)
(2,1) (2,2) (2,3)
(3,1) (3,2) (3,3).

2^9.

Space for scratch work :