Circle the most correct answer.

A mass on a spring moves according to the equation $x'' + 2cx' + \omega_0^2 x = 0$, where $c \geq 0$ is called the *dampening constant*, $\omega_0 > 0$ is called the *spring constant* and $x = 0$ is the *equilibrium point.*

1. (1 point) Suppose the spring has dampening constant 0 and spring constant 1. At time $t = 0$, the mass is moved 2 units to the right of the equilibrium point, and then thrown rightwards at a speed of 1. What is the maximum distance from the equilibrium point that the mass will achieve?

   just b/c must be >2 and there is a max dist

   A. 2

   B. $\sqrt{2}$

   C. $\sqrt{5}$

   D. There is no maximal distance.

   $x(0) = 2$, $x'(0) = 1$, $x'' + x = 0$, $x = A\cos(t + \phi)$, $2 = A\cos(\phi)$, $1 = -A\sin(\phi)$

2. (1 point) Suppose the spring has damping constant 2 and spring constant $\sqrt{3}$. At time $t = 0$, the mass is released from a distance of 1 unit to the left of the equilibrium point with a velocity of 4 rightward. How long will it take for the to mass return to the equilibrium point?

   A. $\ln(3)$

   B. $\ln(\sqrt{3})$

   C. $\frac{3}{2}$

   D. The mass will never reach the equilibrium point.

   $x'' + 4x' + 3x = 0$, $x(0) = -1$, $x'(0) = 4$

   $t = 3t + \ln 3$, $2t = \ln 3$, $t = \ln\sqrt{3}$

3. (1 point) Assume the function $y(x)$ solves $y'' - y = 0$ with $y(0) = 0$ and $y'(0) = 1$. What is $y(1)$?

   A. $\frac{1}{2e}(e^2 - 1)$

   B. $e^{-1} - e$

   C. $\frac{1}{2}$

   D. $\frac{1}{2}e^2$

4. (1 point) Assume the function $y(t)$ solves $y'' - y = e^{2t}$ with $y(0) = 0$ and $y'(0) = 1$. What is $y(1)$?

   A. $\frac{1}{6e}(1 - e^2 + e^3)$

   B. $\frac{1}{10e}(1 - 3e^2 + 2e^3)$

   C. $\frac{1}{3e}(e^3 - e^2 - 1)$

   D. $\frac{1}{3e}(e^3 - 1)$

   $3ae^{2t} = e^{2t}$, $a = \frac{1}{3}$

5. (1 point) Assume the function $y(x)$ solves $y'' + y = 0$ with $y(0) = 0$ and $y'(0) = 2$. What is $y(\pi)$?

   A. 0

   B. $\frac{1}{2}(e^{\pi} - e^{-\pi})$

   C. -2

   D. 2

$y''+y=0 \quad \lambda = \pm \frac{\sqrt{-4}}{2} = \pm i$

$y_1 = \cos t \quad y_2 = \sin t$

$C_1 = 0 \quad C_2 = 2$

$y = 2\sin t$

$y(\pi) = 0$

6. (1 point) Assume the function $y(x)$ solves $y'' + y = \sec(x)$ with $y(0) = 0$ and $y'(0) = 1$. What is ~~$y(1)$~~? $y(\frac{\pi}{2})$

   A. $\sqrt{2}\ln(2) + \pi$

   B. $\frac{1}{\sqrt{2}}(\ln(2) + \pi)$

   C. $1 + \frac{\pi}{2}$

   D. $\frac{1}{4\sqrt{2}}(\pi + 4 - 2\ln(2))$

$y_1 = \cos t \quad y_2 = \sin t$

$W = \cos^2 + \sin^2 = 1$

$v_1 = -\int \frac{1}{\cos t} \cdot \sin t \, dt$ let $u = \cos$

$v_1 = \int \frac{1}{u} \, du = \ln|\cos t|$

$v_2 = \int \frac{1}{\cos t} \cdot \cos t \, dt = t$

$y = \cos t \ln|\cos t| + t\sin t$

$0 = \ln|1| + 0 + 0 + C$

$C_1\cos t + C_2\sin t$

$1 = C_2$

$y = \cos t \ln|\cos t| + \ldots$

$y(\frac{\pi}{2}) = 0 + \frac{\pi}{2}$

7. (1 point) Assume the function $y(t)$ solves $y'' - 2y' + y = 0$ with $y(0) = 0$ and $y'(0) = 1$. What is $y(2)$?

   A. $2e^2$

   B. $2e + 2$

   C. e^2

   D. $3e^2$

$y''-2y'+y=0 \quad \lambda = \frac{2 \pm \sqrt{4-4}}{2} = 1$

$y_1 = e^t \quad y_2 = te^t$

$0 = C_1$

$1 = C_1 + C_2 + 0 \quad y = te^t$

$C_2 = 1 \quad y(2) = 2e^2$

8. (1 point) Assume the function $y(t)$ solves $y'' - 2y' + y = t$ with $y(0) = 0$ and $y'(0) = 1$. What is $y(1)$?

   A. $e + 3$

   B. $2e + 2$

   C. $e + 2$

   D. 3

$y_1 = e^t \quad y_2 = te^t$

$at + b = y_p$

$y_p' = a \quad -2a + at + b = t$

$y_p'' = 0 \quad a = 1 \quad b = 2$

$y = t + 2 + c_1e^t + c_2te^t$

$0 = 2 + C_1 \quad C_1 = -2$

$1 = 1 + C_1 + C_2$

$C_2 = 2$

$y(1) = 3$

9. (2 points) Consider the equation $t^2y'' - 2y = 0$. One solution to this equation is t^2. Suppose that $y(t)$ is another solution such that $y(1) = 1$ and $y'(1) = -1$. What is $y(2)$?

   A. $e^{\sqrt{2}} + e^{-\sqrt{2}}$

   B. 4

   C. $\frac{9}{2}$

   D. $\frac{1}{2}$

just b/c deriv was negative so expect to be < 1