Name: ______________________________

UCLA ID number: ______________________

Discussion section letter: _____

Math 33B: Differential equations

First test

Instructor: Preston Wake

Winter 2016

Separable

Linear

Exact

Page	Points	Score
1	5	5
2	5	3
3	5	5
Total:	15	13

$(y^2+1)^{\frac{3}{2}} = 6x^2$

$\left((y^2+1)^{\frac{3}{2}}\right)^{\frac{2}{3}} = (6)^{2/3}$

$y(0)=1$

$yy' = 4x\sqrt{y^2+1}$

$\int \frac{y}{\sqrt{y^2+1}}\,dy = \int 4x\,dx$

$\frac{1}{2}\int \frac{du}{\sqrt{u}} = 2x^2 + C$

$u = y^2+1$

$du = 2y\,dy$

$\frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right) = 2x^2 + C$

$\frac{1}{3}(y^2+1)^{\frac{3}{2}} = 2x^2 + C$

$C = \frac{\frac{1}{3}(y^2+1)^{\frac{3}{2}}}{2x^2}$

$C = \frac{\frac{1}{3}(1+1)^{3/2}}{0}$

$C \neq 0$

$\frac{\frac{1}{3}(2+1)^{\frac{3}{2}}}{1}$

$2^{3}\sqrt{8} = 2$

Mark each statement as true or false.

1. (1 point) Every linear equation is seperable.
   ○ True ● False

2. (1 point) The function $y = e^{2t}$ solves the equation $(y')^2 = y^2$
   ○ True ● False

3. (1 point) If x and y are functions of t, then $\frac{d}{dt}F(x, y) = \partial_x F \cdot x' + \partial_y F \cdot y'$.
   ● True ○ False

4. (1 point) Suppose that y solves the equation $y' = y\sin(t^2)$ and that $y(0) > 0$. Then $y(t) > 0$ for all t where y is defined.
   ● True ○ False

5. (1 point) The equation $P(x, y)dx + dy = 0$ is exact if and only if $P(x, y)$ depends only on x.
   ● True ○ False

$P = \partial_x F$

$Q = \partial_y F$

$P\,dx + Q\,dy = 0$

$\partial_x F\, x' + \partial_y F\, y' = 0$

$\partial_x(Q) = 0$

$\partial_y(P) = 0$ so [illegible]

Circle the most correct answer.

6. (1 point) Which of these functions is an integrating factor for the equation $y' - \sin(t)y = 2te^t$?

   A. e^{-t}

   B. $\cos(t)$

   C. $e^{-\cos(t)}$

   D. $e^{\cos(t)}$

$y' = \sin(t)y + 2te^t$

$a(t) = \sin(t)$

$A(t) = \int a(t)\,dt = -\cos(t)$

$e^{-A(t)}$ = int. factor

$e^{-(-\cos(t))}$

$e^{\cos(t)}$

7. (1 point) Assume the function $y(x)$ solves $y' + \frac{2y}{x} = \frac{\cos(x)}{x^2}$ with $y(\pi) = 1$. What is $y(\frac{\pi}{2})$?

   A. $\frac{4}{\pi^2}$

   B. 4

   C. $\frac{4}{\pi^2} + 4$

   D. 0

$y' = -\frac{2}{x}y + \frac{\cos(x)}{x^2}$

$-\frac{2}{x} = a(x)$

$A(x) = \int a(x)\,dx = -2\ln(x)$

$e^{+2\ln(x)} = x^2$

$\frac{d}{dx}(yx^2) = 2xy + x^2y'$

$\frac{d}{dx}(yx^2) = x^2\left(\frac{2y}{x} + y'\right)$

$\cos(x)$

$yx^2 = \int \cos(x)\,dx$

$yx^2 = \sin(x) + C$

$C = \frac{yx^2}{\sin x}$

$C = \frac{1 \cdot \pi^2}{\sin(\pi)}$ 0

$C = 0$

$y = \frac{\sin(x)}{x^2}$

$y = \frac{\sin(\pi/2)}{\left(\frac{\pi}{2}\right)^2}$

$= \frac{4}{\pi^2}$

8. (1 point) Assume the function $y(x)$ solves $xy' = (1+2x^2)y$ with $y(1) = 1$. What is $y(2)$?

   A. $2e^4$

   B. $-2e^3$

   C. $\ln(2) + 4 + e^{-1}$

   D. $2e^3$

$\int \frac{dy}{y} = \int \left(\frac{1}{x} + 2x\right) dx$

$\ln(y) = \ln(x) + x^2 + C$

$y = Ae^{\ln(x) + x^2}$

$y = Axe^{x^2}$ $\quad 1 = A(1)e^1$

$y = e^{-1}xe^{x^2}$

$y = e^{-1}(2)e^{(2)^2}$

$y = 2e^{-1}e^4$

$y = 2e^3$

9. (1 point) Assume the function $y(x)$ solves $y' + y = 2e^{-2x}$ with $y(0) = 0$. What is $y(1)$?

   A. $\frac{2}{e^2}(e-1)$

   B. $-2e^{-2}$

   C. $2e^{-2} + 2e^{-1}$

   D. No such function y exists.

$y' = -y + 2e^{-2x}$

$-1 = a(x)$

$A(x) = -x$

$\frac{d}{dx}(ye^x) = e^xy + e^xy'$

$e^x(y' + y)$

$ye^x = \int e^x(2e^{-2x})\,dx$

$ye^x = \int 2e^{-x}\,dx$

$ye^x = -2e^{-x} + C$

$ye^x + 2e^{-x} = C$

$0 + 2e^0 = C$

$C = 2$

$ye^x + 2e^{-x} = 2$

$ye + \frac{2}{e} = 2$

$\frac{ye}{e} = \frac{2 - \frac{2}{e}}{e}$

$y = 2e^{-1} - 2e^{-2}$

$= 2e^{-1}(1 - e^{-1})$

$= 2e^{-2}(e - 1)$

10. (1 point) Assume the function $y(x)$ solves $yy' = 4x\sqrt{y^2+1}$ with $y(0) = 1$. What is $y(1)$?

   A. $\sqrt{2+\sqrt{2}-1}$

   B. $\sqrt{5+4\sqrt{2}}$

   C. $-\sqrt{5+4\sqrt{2}}$

   D. $\sqrt{2}$

11. (2 points) Which of the following equations defines an implicit solution to the differential equation $y' = -\frac{y}{x+y}$?

A. $x^3 + y^2 = C$

B. $xy^3 = C$

C. $x^2y = C$

D. $2xy + y^2 = C$

$y'(x+y) = -y$

$y + (x+y)y' = 0$

$(y)dx + (x+y)dy = 0$

$\partial_y P = \partial_y(y) = 1$

$\partial_x Q = \partial_x(x+y) = 1$

$\partial_x F = P$

$F = \int y\,dx + \phi(y)$

$F = yx + \phi(y)$

$\partial_y F = Q$

$\partial_y(yx + \phi(y)) = x + y$

$x + \phi'(y) = x + y$

$\phi'(y) = y$

$\phi(y) = \int y\,dy = \frac{1}{2}y^2$

$F = C = xy + \frac{1}{2}y^2$

$= 2xy + y^2$

12. (3 points) Assume the function $y(x)$ solves satisfies the equation $\left(\sqrt{x} + \frac{y}{\sqrt{x}}\right)dx + 2\sqrt{x}dy = 0$ with $y(1) = 1$. What is $y(4)$?

A. 2

B. $-\frac{2}{3}$

C. 0

D. $-\frac{4}{3}$

$\partial_y\left(\sqrt{x} + \frac{y}{\sqrt{x}}\right) = 0 + \frac{1}{\sqrt{x}}$

$\partial_x(2x^{\frac{1}{2}}) = 2(\frac{1}{2}x^{-\frac{1}{2}}) = \frac{1}{\sqrt{x}}$ exact

$\partial_x F = P$ $\quad \partial_y F = Q$

$F = \int 2\sqrt{x}\,dy + \phi(x)$

$F = 2\sqrt{x}y + \phi(x)$

$\partial_x(2yx^{\frac{1}{2}} + \phi(x)) = \sqrt{x} + \frac{y}{\sqrt{x}}$

$\frac{y}{\sqrt{x}} + \phi'(x) = \sqrt{x} + \frac{y}{\sqrt{y}}$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

$\phi'(x) = \sqrt{x}$

$\phi(x) = \int x^{\frac{1}{2}}dx$

$= \frac{2}{3}x^{\frac{3}{2}}$

$C = 2\sqrt{1}(1) + \frac{2}{3}(1)^{\frac{3}{2}}$

$C = 2 + \frac{2}{3}$

$C = \frac{8}{3}$

$C = F = 2\sqrt{x}y + \frac{2}{3}x^{\frac{3}{2}}$

$\frac{8}{3} = 2\sqrt{4}(y) + \frac{2}{3}(4)^{3/2}$

$y^{3} = \sqrt{64} = 8 \cdot \frac{2}{1} = \frac{16}{3}$

$\frac{8}{3} = 4y + \frac{16}{3}$

$-\frac{16}{3} \quad -\frac{16}{3}$

$\frac{-8}{3} = \frac{4y}{4}$

$\frac{-8}{3} \cdot \frac{1}{4} \qquad -\frac{2}{3}$

$\partial_x F = P \quad \partial_y F = Q$

$F = \int P\,dx + \phi(y)$

$\partial_y F = Q$

$\partial_y\left(\int P\,dx + \phi(y)\right) = Q$

$\frac{\partial P}{\partial y}(x,y)dx + \phi'(y) = Q$