Mark each statement as true or false.

1. (1 point) Every linear equation is seperable. separable is a special case in linear
   - ○ True ⊗ False

2. (1 point) The function $y = e^{2t}$ solves the equation $(y')^2 = y^2$ ← $y' = 2e^{2t}$ $4e^{4t} \neq e^{4t}$
   - ○ True ⊗ False

3. (1 point) If x and y are functions of t, then $\frac{d}{dt}F(x, y) = \partial_x F \cdot x' + \partial_y F \cdot y'$. ← chain rule
   - ⊗ True ○ False

4. (1 point) Suppose that y solves the equation $y' = y\sin(t^2)$ and that $y(0) > 0$. Then $y(t) > 0$ for all t where y is defined. $y=0$ solves so that is asymptote
   - ⊗ True ○ False

5. (1 point) The equation $P(x, y)dx + dy = 0$ is exact if and only if $P(x, y)$ depends only on x.
   - ⊗ True ○ False

$F_x = P(x,y)$ $F_y = 1$

$F = \int P(x,y)dx$ $F = y + C$

cannot include a y and still satisfy our other partial

Circle the most correct answer.

6. (1 point) Which of these functions is an integrating factor for the equation $y' - \sin(t)y = 2te^t$?

   A. e^{-t}

   B. $\cos(t)$

   C. $e^{-\cos(t)}$

   D. $e^{\cos(t)}$

$\mu = e^{-\int \sin(t)\,dt}$

$\boxed{\mu = e^{\cos t}}$

7. (1 point) Assume the function $y(x)$ solves $y' + \frac{2y}{x} = \frac{\cos(x)}{x^2}$ with $y(\pi) = 1$. What is $y(\frac{\pi}{2})$?

   A. $\frac{4}{\pi^2}$

   B. 4

   C. $\frac{4}{\pi^2} + 4$

   D. 0

$\mu = e^{\int \frac{2}{x}dx}$ $\mu = x^2$

$x^2 y = \int \left(\frac{\cos x}{x^2}\right) x^2\, dx$ $x^2 y = \sin x + C$

$y = \frac{\sin x + C}{x^2}$

$1 = \frac{\sin \pi + C}{\pi^2}$

$1 = \frac{C}{\pi^2}$

$C = \pi^2$

$y(\frac{\pi}{2}) = \frac{\sin\frac{\pi}{2} + \pi^2}{(\frac{\pi}{2})^2}$

$y(\frac{\pi}{2}) = \frac{(1+\pi^2)4}{\pi^2}$

8. (1 point) Assume the function $y(x)$ solves $xy' = (1+2x^2)y$ with $y(1) = 1$. What is $y(2)$?

   A. $2e^4$

   B. $-2e^3$

   C. $\ln(2) + 4 + e^{-1}$

   D. $2e^3$

$\frac{1}{y}dy = \frac{1+2x^2}{x}dx$

$\ln|y| = \ln|x| + x^2 + C$

$y = x e^{x^2} e^C$

$1 = 1e^1e^C$

$e^C = e^{-1}$

$y = xe^{x^2-1}$

$\boxed{y(2) = 2e^3}$

9. (1 point) Assume the function $y(x)$ solves $y' + y = 2e^{-2x}$ with $y(0) = 0$. What is $y(1)$?

   A. $\frac{2}{e^2}(e-1)$

   B. $-2e^{-2}$

   C. $2e^{-2} + 2e^{-1}$

   D. No such function y exists.

$\mu = e^{\int 1\,dx} = e^x$

$ye^x = \int 2e^{-2x}e^x\,dx$

$ye^x = -2e^{-x} + C$

$y = \frac{-2e^{-x} + C}{e^x}$

$0 = \frac{-2e^0 + C}{e^0} = \frac{-2+C}{1}$ $C=2$

$y = -2e^{-2x} + 2e^{-x}$

$y(1) = -2e^{-2} + 2e^{-1}$

$= \frac{2}{e^2}(e-1)$

10. (1 point) Assume the function $y(x)$ solves $yy' = 4x\sqrt{y^2+1}$ with $y(0) = 1$. What is $y(1)$?

   A. $\sqrt{2+\sqrt{2}} - 1$

   B. $\sqrt{5+4\sqrt{2}}$

   C. $-\sqrt{5+4\sqrt{2}}$

   D. $\sqrt{2}$

$\frac{y}{\sqrt{y^2+1}}dy = 4x\,dx$

$(y^2+1)^{1/2} = 2x^2 + C$

$y^2+1 = 4x^4 + 4x^2C + C^2$

$y^2 = 4x^4 + 4x^2C + C^2 - 1$

$y = \pm\sqrt{4x^4 + 4x^2C + C^2 - 1}$

$1 = \pm\sqrt{0+0+C^2-1}$

$C^2 = 2$

$C = \sqrt{2}$

$y(1) = \sqrt{4 + 4\sqrt{2} + 2 - 1}$

$y(1) = \sqrt{5 + 4\sqrt{2}}$

11. (2 points) Which of the following equations defines an implicit solution to the differential equation $y' = -\frac{y}{x+y}$?

A. $x^3 + y^2 = C$

B. $xy^3 = C$

C. $x^2y = C$

D. $2xy + y^2 = C$

$(x+y)y' = -y$

$(x+y)dy = -y\,dx$

$y\,dx + (x+y)dy = 0$

$F_x = y \qquad F_y = x+y$

$F = xy + C_1 \qquad F = xy + \frac{1}{2}y^2 + C_2$

is a different C, but that does not matter

$xy + \frac{1}{2}y^2 + C = 0$ or $2xy + y^2 = C$

12. (3 points) Assume the function $y(x)$ solves satisfies the equation $\left(\sqrt{x} + \frac{y}{\sqrt{x}}\right) dx + 2\sqrt{x}dy = 0$ with $y(1) = 1$. What is $y(4)$?

A. 2

B. $-\frac{2}{3}$

C. 0

D. $-\frac{4}{3}$

$\left(\sqrt{x} + \frac{y}{\sqrt{x}}\right)dx + 2\sqrt{x}\,dy = 0$

$F_x = \sqrt{x} + \frac{y}{\sqrt{x}} \qquad F_y = 2\sqrt{x}$

$F = \frac{2}{3}x^{3/2} + 2x^{1/2}y + C_1 \qquad F = 2x^{1/2}y + C_2$

$\frac{2}{3}x^{3/2} + 2x^{1/2}y = C$

$\frac{2}{3}(1) + 2(1)(1) = C$

$\frac{2}{3} + 2, \; C = \frac{8}{3}$

$\frac{2}{3}(4)^{3/2} + 2(4)^{1/2}y = \frac{8}{3}$

$\frac{2}{3}(8) + 2(2)y = \frac{8}{3}$

$\frac{16}{3} + 4y = \frac{8}{3}$

$4y = -\frac{8}{3}$

$y = -\frac{2}{3}$