

Part I: Multiple choice. Please write your answers (A, B, C, ...) in the boxes on the right.

1. (3 points) Determine values of the constants a and b that make the differential equation exact. (1)

$$(3x^2y + ax^5y^3) dx + (bx^3 + x^6y^2) dy = 0$$

P Q

- A. $a = 2, b = -1$
- B. $a = 1, b = 2$
- C. $a = 2, b = 1$
- D. $a = -1, b = 2$
- E. None of the above.

$$\frac{\partial Q}{\partial x} = 3bx^2 + 6x^5y^2$$

$$\frac{\partial P}{\partial y} = 3x^2 + 3x^6y^2a$$

C

2. (4 points) Which of the following integrating factors is suitable for the differential equation

$$(x+2) \sin y dx + x \cos y dy = 0? \quad (2)$$

- A. $x e^x$
- B. $1 + \frac{1}{x}$
- C. $e^{\cos x}$
- D. $\sin x$
- E. None of the above.

$$\frac{\partial Q}{\partial x} = \cos y \quad \frac{\partial P}{\partial y} = (x+2) \cos y$$

$$\frac{1}{Q} \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right)$$

$$= \frac{1}{x \cos y} ((x+2) \cos y - \cos y)$$

$$= \frac{1}{x \cos y} (x+1 \cos y)$$

$$= \int \frac{x+1}{x} dx$$

$$= e^{\int \frac{1}{x} dx} = e^{\ln x}$$

E