Math 33B Quiz 2 (Thursday) Fall 2018

Name ______________________ TA section: 2. B

UID ______________________

- You have 10 minutes
- No calculators
- Show sufficient work

1. Find the solution of the initial value problem. Discuss the interval of existence and provide a sketch of your solution.

$$xy' + 2y = \sin x, \qquad y\left(\frac{\pi}{2}\right) = 0$$

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$xy' + 2y = \sin(x)$

$y' = -\frac{2y}{x} + \frac{\sin x}{x}$

Integrating Factor: $\mu = e^{-\int a(x)dx} = e^{+\int \frac{2}{x} dx} = e^{2\ln|x|} = x^2$

Integr. by parts:

$u = x$ $\quad v = -\cos x$

$du = 1\,dx$ $\quad dv = \sin x\, dx$

$(\mu y)' = \mu f$; $\int (x^2 y)' = \int \frac{x^2 \sin x}{x}$

$\int (x^2 y)' = \int \frac{x \sin x}{\cancel{x}} dx$

$x^2 y = -x\cos x - \int -\cos x \, dx$

$x^2 y = -x\cos x + \int \cos x \, dx = -x\cos x + \sin x + C$

$y = \frac{-x\cos x}{x^2} + \frac{\sin x}{x^2} + \frac{C}{x^2} = \frac{-\cos x}{x} + \frac{\sin x}{x} + \frac{C}{x^2}$

$y(\pi/2) = 0$;

$0 = \cancel{\frac{0-\cos(\pi/2)}{\pi/2}} + \frac{\sin(\pi/2)}{(\pi/2)^2} + \frac{C}{(\pi/2)^2}$ went wrong. forgot to square denom.

$0 = \frac{4}{\pi^2} + \frac{4C}{\pi^2}$

$\frac{-2\pi}{-4} = \frac{4C}{\pi^2}$ $\quad C = 1$

$C = -1$ noice

$$\boxed{y = \frac{-\cos x}{x} + \frac{\sin x}{x} - \frac{1}{x^2}}$$

The solution exists over the interval $(0, \infty)$, since $\pi/2$ is in the interval.