33B Midterm 2

[redacted]

TOTAL POINTS

34 / 40

QUESTION 1

auto. Equation 11 pts

1.1 Phase Line 3 / 3

✓ **- 0 pts** Correct

- 1 pts arrows missing/wrong

- 1 pts graph wrong somewhere

- 1 pts missing scale

- 3 pts not the phase line

- 1 pts too many zero's in the graph

1.2 Eq. Solutions 3 / 3

✓ **- 0 pts** Correct

- 1 pts -3 no conclusion

- 1 pts 2 is stable

- 1 pts 5 is unstable

1.3 Graph sketch 2 / 2

✓ **- 0 pts** Correct

- 2 pts no solution/wrong solution

- 1 pts graph between -3 and 2 wrong/missing

- 1 pts graph between 2 and 5 wrong/missing

1.4 particular solution 3 / 3

✓ **+ 3 pts** Correct

+ 1 pts No

+ 1 pts Uniqueness theorem can be applied

+ 0 pts wrong/no answer

+ 1 pts cannot cross the equilibrium solution y(t) = 2

QUESTION 2

Existence and Uniqueness 8 pts

2.1 Apply? Rectangle? 5 / 5

✓ **+ 2 pts** continuous

✓ **+ 2 pts** derivative continuous

✓ **+ 1 pts** rectangle

+ 0 pts no points

2.2 x_0(2)=5? 1 / 3

✓ **+ 1 pts** Correct

+ 2 pts justification

+ 0 pts no points

It's not possible that x0(2) = 5 since the solution must be increasing

QUESTION 3

3 Particular Solution 6 / 6

✓ **- 0 pts** Correct

- 1 pts Mixed up a minus sign

- 3 pts Didn't try the right guess (ae^3t)

- 6 pts Didn't attempt method of undetermined coefficients.

- 1 pts Incorrect division

- 1 pts Incorrect multiplication

- 1 pts Put constant in solution

- 3 pts Forgot to include an undetermined coefficients in MOC.

QUESTION 4

2. order equation constant coefficients 5 pts

4.1 verify solutions 1 / 3

- 0 pts Correct

✓ **- 2 pts Didn't explicitly check boundary conditions**

- 1 pts Only checked one boundary condition

- 1 pts Didn't correctly check that they satisfy the ODE.

4.2 existence and uniqueness? 0 / 2

- 0 pts Correct

- 2 pts Didn't understand that solution was non-unique.

✓ **- 2 pts Didn't state that the boundary conditions being defined at different times renders the existence and uniqueness theorem irrelevant.**

- **1 pts** Not clear if you actually meant that the "initial" conditions are defined at different times.

QUESTION 5

2. order equation 7 pts

5.1 verify solutions **4 / 4**

✓ **- 0 pts Correct**

- 2 pts incorrect calculation

- 4 pts incorrect calculation

5.2 fundamental set **3 / 3**

✓ **- 0 pts Correct**

- 1 pts conclusion is incorrect,

- 1 pts some work, calculation incorrect,

- 3 pts conclusion incorrect, wrong calculation

- 2 pts some work

QUESTION 6

6 planar system **3 / 3**

✓ **- 0 pts Correct**

- 2 pts incorrect, but some work

- 1 pts minor mistake

- 3 pts no work

65

MIDTERM 2

11/16/2018

Math33B
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Name: [redacted]

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section: D

Problem	Points	Score
1	11	
2	8	
3	6	
4	5	
5	7	
6	3	
Total	40	

Instructions

(1) This exam has 6 problems. Make sure you have all pages.
(2) Enter your name, SID number, and discussion section on the top of this page.
(3) **Use a PEN** to record your final answers.
(4) If you need **more space**, use the extra page at the end of the exam.
(5) NO Calculators, computers, books or notes of any kind are allowed.
(6) Show your work. Unsupported answers will not receive full credit.
(7) Good Luck!

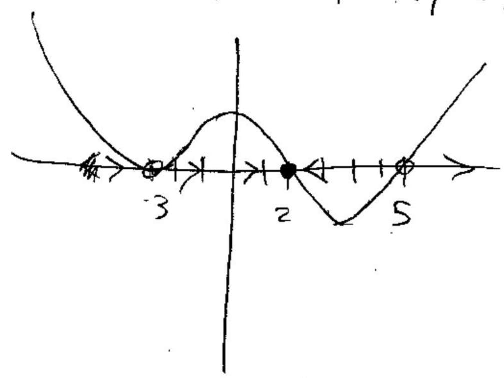
Exercise 1. (11pt)
Consider the autonomous first-order differential equation.

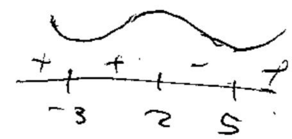
$$y' = (y+3)^2(y-2)(y-5)$$

(1) Draw the phase line. (3pt)

$y=-3,\quad y=2 \quad y=5$

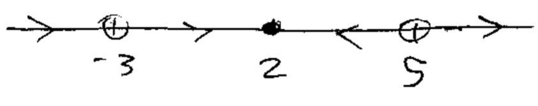
$f(y) = (y+3)^2(y-2)(y-5)$





number line to help plot

phase ~~diagram~~ line



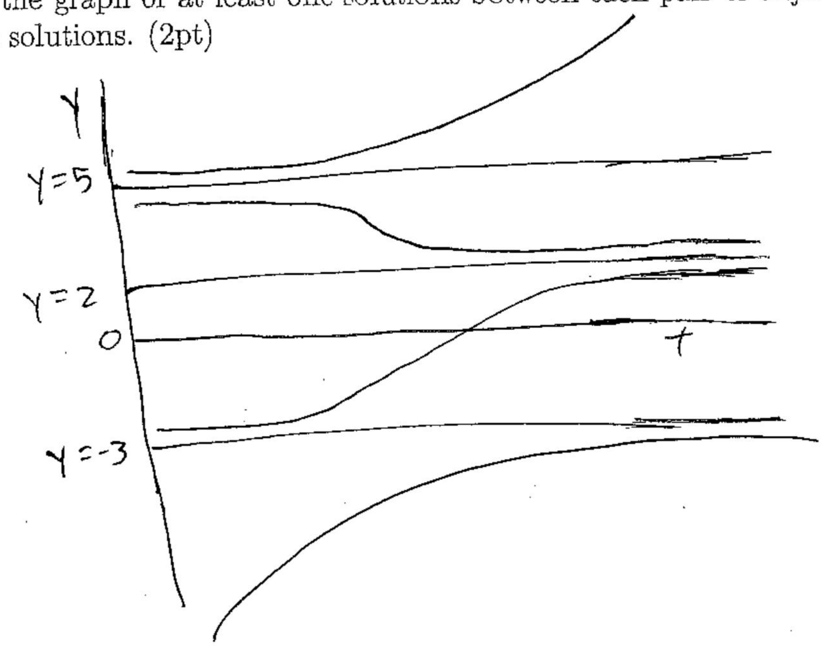
(2) What are the equilibrium solutions? Which are stable, and which are unstable? (3pt)

$y = -3$ undetermined

$y = 2$ stable

$y = 5$ unstable

(3) Sketch the graph of at least one solutions between each pair of adjacent equilibrium solutions. (2pt)



(4) Let $y_p(t)$ be a particular solution to the equation which satisfies $y_p(0) = 0$. Is it possible that $y_p(2) = 2$? Justify your answer. (3pt)

No, because solutions are unique and do not ever cross, and ~~sen~~ since y = 2 is already a solution, yp(t) will never be equal to 2 but instead ~~approaches~~ can only approach values very close to 2.

Exercise 2. (8pt) Consider the differential equation

$$\frac{dx}{dt} = \frac{\sqrt{x^2-4}}{t^2}$$

(1) Can you apply the existence and uniqueness theorem to the initial value problem $x_0(1) = 6$? Justify your answer and give the biggest rectangle in which you can apply it containing the given point (if it exists). (5 pt)

$\frac{1}{t^2}(x^2-4)^{1/2}$

$\frac{2x}{2t^2}(x^2-4)^{-1/2}$

$t=1$

$x=6$

$f(x,t) = \frac{\sqrt{x^2-4}}{t^2}$

$x^2 - 4 \geq 0$

$x^2 \geq 4$

$x \geq 2, \quad x \leq -2$

$t \neq 0$

$\frac{\partial f}{\partial x} = \frac{x}{t^2\sqrt{x^2-4}}$

$x > 2, \quad x < -2$

$t \neq 0$

$f(x,t)$ continuous on $x: (-\infty, -2] \cup [2, \infty)$
$t: (-\infty, 0) \cup (0, \infty)$

$\frac{\partial f}{\partial x}$ continuous on $x: (-\infty, -2) \cup (2, \infty)$
$t: (-\infty, 0) \cup (0, \infty)$

Since $f(x,t)$ and $\frac{\partial f}{\partial x}$ are continuous at $t_0 = 1$ and $x_0(1) = 6$, there exists a unique solution because the criteria for the existence and unique theorem applies. Thus, the biggest rectangle in which you can apply it is for $2 < x < \infty$, $0 < t < \infty$.

(2) Can $x_0(2) = 5$ ($x_0(t)$ is the solution to the initial value problem in part 1))?(1pt) Justify your answer. (2pt)

No, $x_0(2) \neq 5$. This is because since the unique solution $x_0(1) = 6$ and therefore $x(t) \geq 6$, exists, it cannot cross this value, so ~~[illegible]~~ $x_0(2)$ has to be greater than 6.

Exercise 3. (6pt) Find a particular solution to the following differential equation

$$3y'' + 2y' - y = -4e^{3t}.$$

Guess: $y = Ae^{3t}$ $\cancel{\text{fou}}$ $y' = 3Ae^{3t}$ $y'' = 9Ae^{3t}$

$$3(9Ae^{3t}) + 2(3Ae^{3t}) - Ae^{3t} = -4e^{3t}$$

$$27Ae^{3t} + \cancel{2}\, 6Ae^{3t} - Ae^{3t} = -4e^{3t}$$

$$32Ae^{3t} = -4e^{3t}$$

$$32A = -4$$

$$A = -\frac{1}{8}$$

$$\boxed{y_p = -\frac{1}{8}e^{3t}}$$

Exercise 4. (5pt) Consider the following problem:

$$y'' + y = 0 \quad y(0) = 0 \;\; y'(\pi/2) = 0$$

(1) Show that $y(t) = C \cdot \sin(t)$ is a solution for any constant C. (3pt)

$$y'(t) = C\cos(t)$$
$$y''(t) = -C\sin(t)$$

$$y'' + y = 0$$
$$-C\sin(t) + C\sin(t) = 0$$
$$0 = 0 \checkmark$$

$\therefore$ $y(t) = C\sin(t)$ is a solution for any constant C.

(2) Why does this not violate the 2. order existence and uniqueness theorem? (2pt)

This does not violate the 2nd order existence and uniqueness theorem because $y(t)$ is a trignometric function and does not follow the 2nd order linear equation model.

Exercise 5. (8pt) Consider the differential equation

$$y'' + \frac{1+x}{x}y' - \frac{1}{x}y = 0$$

(1) Check that $1+x$ and $\frac{2x^2+6x+4}{x+2}$ are solutions to the above equation. (4pt)

$y = 1+x \qquad y' = 1 \qquad y'' = 0$

$$0 + \frac{1+x}{x}(1) - \frac{1+x}{x} = 0$$

$$\frac{1+x}{x} - \frac{1+x}{x} = 0$$

$$0 = 0 \checkmark$$

$\therefore$ $1+x$ is a solution.

$$y = \frac{2x^2+6x+4}{x+2}$$

$$= \frac{(2x+2)\cancel{(x+2)}}{\cancel{x+2}}$$

$y = 2x+2 \qquad y' = 2 \qquad y'' = 0$

$$0 + \frac{1+x}{x}(2) - \frac{1}{x}(2x+2) = 0$$

$$\frac{2x+2}{x} - \frac{2x+2}{x} = 0$$

$$0 = 0$$

$\therefore$ $\frac{2x^2+6x+4}{x+2}$ is a solution.

(2) Do they form a fundamental set of solutions?(1pt) Justify your answer. (2pt)

$y_1 = x+1$

$y_2 = \frac{2x^2+6x+4}{x+2}$

$= 2x+2$

No they do not ~~[illegible]~~ because there is a constant ~~[illegible]~~ $c = \frac{1}{2}$ s.t. ~~[illegible]~~ $y_1 = \frac{1}{2} y_2$. ~~[illegible]~~ It follows that the Worzkian is also 0, showing that the solutions are lineарly dependent and ∴ cannot form a fundamental set of solutions

forgot spelling ✓

$$W = \det \begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix} = 0$$

$$y_1 y_2' - y_1' y_2 = 0$$

$$(x+1)(2) - 1(2x+2) = 0$$

$$2x+2-2x-2=0$$

$$0=0$$

∴ the solutions are lineerly dependent~~[illegible]~~ and do not form a fundamental set of solutions

$$y_1 = \frac{1}{2} y_2$$

$$x+1 = \tfrac{1}{2}(2x+2)$$

$$x+1 = x+1$$

Exercise 6. (3pt)
Consider the second order equation
$$y'' - 2e^t y' - \tan(t)y = \sqrt{t^2+1}.$$
Write this equations as a planar system of first-order equations.

$v = y'$
$v' = y''$

$v' - 2e^t v - \tan(t)y = \sqrt{t^2+1}$
$v' = 2e^t v + \tan(t) y + \sqrt{t^2+1}$

$$\boxed{\begin{aligned} v &= y' \\ v' &= 2e^t v + \tan(t)y + \sqrt{t^2+1} \end{aligned}}$$

Extra page

Extra page