

33B midterm 1

TOTAL POINTS

37 / 40

QUESTION 1

integration factor 8 pts

1.1 integration factor 4 / 4

- ✓ - 0 pts Correct
 - 1 pts minor mistake
 - 4 pts no work
 - 3 pts subtle work, try to find $h(x)$ but equation incorrect
 - 2 pts get $h(x)$, but not $u(x)$
 - 2 pts get $u(x)$ but without details; know how to get $u(x)$ but calculate incorrectly

1.2 solve 4 / 4

- ✓ - 0 pts Correct
 - 1 pts solution should be in form of $F(x,y) = c$
 - 4 pts no work
 - 3 pts know need to do partial integration, but incorrect.
 - 2 pts correct form $F = \phi + xxx$, but ϕ incorrect; or the other way around.
 - 1 pts minor mistake

QUESTION 2

separable eon 12 pts

2.1 explicit solution 5 / 5

- ✓ + 1 pts Separating the Equation
- ✓ + 1 pts Partial Fractions
- ✓ + 1 pts Computing Integral
- ✓ + 1 pts Log Rule Application
- ✓ + 1 pts Computing Solution
 - + 2 pts Bernoulli Transformation
 - + 1 pts Integrating Factor
 - + 2 pts Rest of Bernoulli Solution
 - + 0 pts No points

2.2 $y(1) = 2 \frac{2}{2}$

- ✓ + 2 pts Correct Answer
 - + 1.5 pts Correct Answer, Wrong Solution
 - + 1 pts Knowing the Process
 - + 0 pts No points

2.3 interval of existence 3 / 3

- ✓ + 1 pts Knowing 0 is not included
 - + 1 pts Correct for their function
- ✓ + 1 pts Correct
- ✓ + 1 pts Knowing 2 is not included.
 - + 0 pts No points

2.4 $y(1) = 0 \frac{1}{2}$

- + 2 pts Correct Answer
- + 1 pts Correct Answer, but on accident
- + 0 pts No points
- + 1 Point adjustment
 - ☞ This is exactly the right idea, except this does not satisfy $y(1)=0$

QUESTION 3

3 mixing problem 6 / 7

- 1 pts Identifying x' = rate in- rate out, rate in = 4
- 2 pts Identify rate out = $x/(50+t)$
- 1 pts Find an integrating factor or homogeneous solution
 - 2 pts Find the general solution
- ✓ - 1 pts Incorporate the initial condition.
 - 0 pts Correct
 - 1 pts Accidentally made equation Homogeneous/ too simple.
 - 1 pts Forgot a factor of 2 in rate out.

QUESTION 4

exact eqn 7 pts

4.1 not exact 3 / 3

✓ - 0 pts Correct

- 3 pts No answer
- 2 pts wrong derivatives
- 1 pts wrong Q derivative
- 3 pts wrong approach
- 1 pts why?
- 1 pts wrong P derivative

4.2 integration factor 4 / 4

✓ - 0 pts Correct

- 1 pts sign mistake
- 3 pts only formula
- 1 pts $a=?$ $b=?$
- 4 pts wrong/no work
- 2 pts right start

QUESTION 5

SA 6 pts

5.1 dir field 3 / 4

- 2 pts No 2. solution
- 2 pts No 1. solution
- ✓ - 1 pts mistake 1. solution
- 1 pts mistake 2. solution
- 4 pts doesn't go through the right points
- 2 pts doesn't go through the right point 1. solution
- + 4 pts correct

5.2 Y/N 2 / 2

- 0.5 pts 1 incorrect
- 1 pts 2 incorrect
- 1.5 pts 3 incorrect
- 2 pts all incorrect
- ✓ + 2 pts correct

MIDTERM 1

10/24/2018

Name:



section:



Math33B

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Problem	Points	Score
1	8	
2	12	
3	7	
4	7	
SA	6	
Total	40	

Exercise 1. (8pt)

Consider the differential equations

$$2y^2 + 4x^2 + 2xy \frac{dy}{dx} = 0$$

- (1) Find the integrating factor for the above equations. (4pt)

(Hint: it only depends on x)

$$(2y^2 + 4x^2) dx + (2xy) dy = 0$$

$$\frac{\partial P}{\partial y} = 4y \quad \frac{\partial Q}{\partial x} = 2y$$

$$h(x) = \frac{1}{Q} \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right) = \frac{1}{2xy} (4y - 2y) = \frac{1}{x} \quad \text{depends only on } x$$

$$\mu(x) = e^{\int h(x) dx} = e^{\ln|x|} = |x|$$

particular: $\mu(x) = x$

- (2) Solve the equation. (4pt)

$$(2y^2 x + 4x^3) dx + 2x^2 y dy = 0$$

$$\frac{\partial P}{\partial y} = 4y x = \frac{\partial Q}{\partial x} = 4xy \quad \therefore \text{exact}$$

$$F_x = P = 2xy^2 + 4x^3$$

$$F(x,y) = x^2 y^2 + x^4 + \phi(y)$$

$$F_y = 2y x^2 + \phi'(y) = Q = 2x^2 y$$

$$\phi'(y) = 0 \Rightarrow \phi(y) = C_0$$

$$F(x,y) = x^2 y^2 + x^4 = C$$

$$\frac{1}{x} (2xy^2 + 4x^3) dx + 2yx^2 dy = 0$$

$$(2y^2 + 4x^2) dx + (2yx) dy = 0$$

Exercise 2. (12pt) Consider the differential equation

$$\frac{dy}{dx} = \frac{y^2 - y}{x}$$

(1) Find the explicit general solution. (5pt)

$$\int \frac{1}{y^2 - y} dy = \int \frac{1}{x} dx \quad y^2 - y \neq 0$$

$$\int \left(\frac{-1}{y} + \frac{1}{y-1} \right) dy = \ln|x| + C_0$$

$$-\ln|y| + \ln|y-1| = \ln|x| + C_1$$

$$\ln \left| \frac{y-1}{y} \right| = \ln|x| + C_1$$

$$\left| \frac{y-1}{y} \right| = e^{\ln|x|} e^C$$

$$1 - \frac{1}{y} = C_2 |x|$$

$$y(x) = \frac{1}{1 - C_2 |x|}$$

$$\boxed{y(x) = \frac{1}{1 + Cx}}$$

$$\frac{1}{y^2 - y} = \frac{A}{y} + \frac{B}{y-1}$$

$$1 = (y-1)A + (y)B$$

$$y=1 \Rightarrow B=1$$

$$y=0 \Rightarrow A=-1$$

(2) Find the solution to this equation that satisfies the initial condition $y(1) = 2$. (2pt)

$$y(1) = 2 = \frac{1}{1 + C(1)} \Rightarrow C = -\frac{1}{2}$$

$$y(x) = \frac{1}{1 - \frac{1}{2}x}$$

$$\boxed{y(x) = \frac{2}{2-x}}$$

(3) What is the interval of existence of the solution you found in (b). (3pt)

$$\text{solution: } x \neq 2$$

$$\text{denominator: } x \neq 0$$

$$\text{point: } x = 1$$

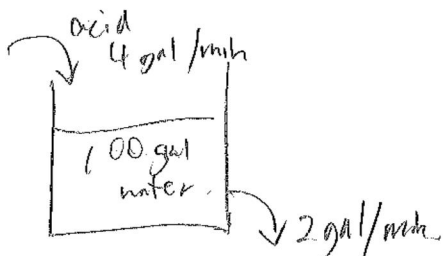
$$\therefore \text{Interval of Existence: } x \in (0, 2)$$

(4) Find the solution to this equation that satisfies the initial condition $y(1) = 0$. (2pt)

$$\boxed{y(x) = 1}$$

$$\frac{dy}{dx} = 0 = \frac{x^2 - 1}{1} = 0$$

Exercise 3. (7pt) Suppose there is a tank filled with 100 gallons of water. Pure acid flows into the tank at a rate of 4 gal/min and the well mixed solution leaves the tank at the rate of 2 gal/min. Let $x(t)$ be the volume in gallons of acid in the tank at time t . Find $x(t)$ for any given time t .



$$x'(t) = 4 - \frac{x}{100+2t} \cdot 2$$

$$x'(t) + \frac{1}{50+t} x = 4$$

$$\mu(t) = e^{\int \frac{1}{50+t} dt} = e^{\ln|50+t|} = |50+t| \stackrel{t > 0}{=} 50+t$$

$$\int (x(50+t))' dt = \int 4(50+t) dt$$

$$x(50+t) = 200t + 2t^2 + C_0$$

$$x(t) = \frac{200t + 2t^2 + C_0}{50+t}$$

Exercise 4. (7pt) Consider

$$(4yx)dx + (5x^2)dy$$

(1) Show that the above equation is not exact. (3pt)

$$p = 4yx \quad Q = 5x^2$$

$$\frac{\partial p}{\partial y} = 4x \neq \frac{\partial Q}{\partial x} = 10x$$

\therefore not exact.

(2) Find a and b such that $x^a y^b$ is an integration factor of the above equation. (4pt)

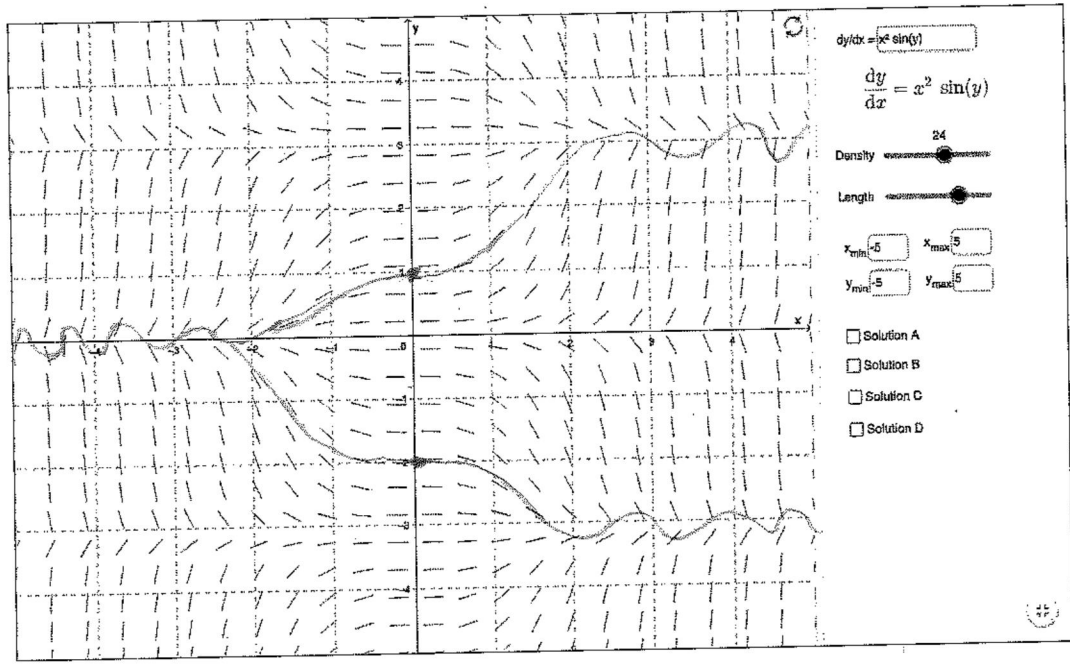
Make separable: $M = \frac{1}{y(x^2)} = y^{-1} x^{-2}$

$$\boxed{a = -1 \quad b = -2}$$

$$\frac{4}{x} dx + \frac{5}{y} dy = 0$$

$$\frac{\partial p}{\partial y} = 0 = \frac{\partial Q}{\partial x} = 0 \quad \text{exact}$$

Field M1 F18.png



1. SHORT ANSWER PROBLEMS

(no explanation needed)

(1) (4pt) Consider the above direction field and draw the solution through (0,1) and the solution through (0,-2).

(2) (2pt) Which of the following are homogeneous differential equations?

- Y N $\sin(\frac{x}{y})dy + 2dx = 0$ $\sin(\frac{x}{y}) = \sin(\frac{x}{y})$ $z = z$ $n = 0$
- Y N $(xy + x^2)dy + (y^2x - x^2y)dx$ $t^2(xy + x^2)$ $t^3(y^2x - x^2y)$ $n_1 \neq n_2$
- Y N $\sin(xy)dy - \cos(xy)dx$ $\sin(t^2(xy))$ $\cos(t^2(xy))$
- Y N $\sqrt{x^2y^2 - 4xy^3}dy + x^2dx$ $t^2 \int (x^2y^2 - 4xy^3)$ $t^2(x^2)$ $n = 2$