33B midterm 1

[redacted]

TOTAL POINTS

34.5 / 40

QUESTION 1

integration factor 8 pts

1.1 integration factor 4 / 4

✓ **- 0 pts Correct**

- 1 pts minor mistake

- 4 pts no work

- 3 pts subtle work, try to find h(x) but equation incorrect

- 2 pts get h(x),but not u(x)

- 2 pts get u(x) but without details; know how to get u(x) but calculate incorrectly

1.2 solve 4 / 4

✓ **- 0 pts Correct**

- 1 pts solution should be in form of F(x,y) = c

- 4 pts no work

- 3 pts know need to do partial integration, but incorrect.

- 2 pts correct form F = \phi + xxxx, but \phi incorrect ; or the other way around.

- 1 pts minor mistake

QUESTION 2

separable eon 12 pts

2.1 explicit solution 5 / 5

✓ **+ 1 pts Separating the Equation**

✓ **+ 1 pts Partial Fractions**

✓ **+ 1 pts Computing Integral**

✓ **+ 1 pts Log Rule Application**

✓ **+ 1 pts Computing Solution**

+ 2 pts Bernoulli Transformation

+ 1 pts Integrating Factor

+ 2 pts Rest of Bernoulli Solution

+ 0 pts No points

2.2 y(1) = 2 2 / 2

✓ **+ 2 pts Correct Answer**

+ 1.5 pts Correct Answer, Wrong Solution

+ 1 pts Knowing the Process

+ 0 pts No points

2.3 interval of existence 1 / 3

+ 1 pts Knowing 0 is not included

+ 1 pts Correct for their function

+ 1 pts Correct

✓ **+ 1 pts Knowing 2 is not included.**

+ 0 pts No points

2.4 y(1) = 0 2 / 2

✓ **+ 2 pts Correct Answer**

+ 1 pts Correct Answer, but on accident

+ 0 pts No points

QUESTION 3

3 mixing problem 6 / 7

- 1 pts Identifying x'= rate in- rate out, rate in = 4

- 2 pts Identify rate out = x/(50+t)

- 1 pts Find an integrating factor or homogeneous solution

- 2 pts Find the general solution

- 1 pts Incorporate the initial condition.

- 0 pts Correct

- 1 pts Accidentally made equation Homogeneous/ too simple.

- 1 pts Forgot a factor of 2 in rate out.

- 1 Point adjustment

turned a 200 into a 150.

QUESTION 4

exact eqn 7 pts

4.1 not exact 3 / 3

✓ **- 0 pts Correct**

- 3 pts No answer

- **2 pts** wrong derivatives
- **1 pts** wrong Q derivative
- **3 pts** wrong approach
- **1 pts** why?
- **1 pts** wrong P derivative

4.2 integration factor **4 / 4**

✓ - **0 pts** **Correct**
- **1 pts** sign mistake
- **3 pts** only formula
- **1 pts** a=? b=?
- **4 pts** wrong/no work
- **2 pts** right start

QUESTION 5

SA 6 pts

5.1 dir field **3 / 4**

- **2 pts** No 2. solution
- **2 pts** No 1. solution

✓ - **1 pts** **mistake 1. solution**
- **1 pts** mistake 2. solution
- **4 pts** doesn't go through the right points
- **2 pts** doesn't go through the right point 1. solution
+ **4 pts** correct

5.2 Y/N **0.5 / 2**

- **0.5 pts** 1 incorrect
- **1 pts** 2 incorrect

✓ - **1.5 pts** **3 incorrect**
- **2 pts** all incorrect
+ **2 pts** correct

MIDTERM 1

10/24/2018

Math33B
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section: D

Problem	Points	Score
1	8	
2	12	
3	7	
4	7	
SA	6	
Total	40	

Exercise 1. (8pt)
Consider the differential equations

$$2y^2 + 4x^2 + 2xy\frac{dy}{dx} = 0$$

(1) Find the integrating factor for the above equations.(4pt)
(Hint: it only depends on x)

$P(x,y)$ $\quad$ $Q(x,y)$
$(2y^2 + 4x^2)\,dx + 2xy\,dy = 0$

$$g = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$$
$$= \frac{1}{2xy}(4y - 2y)$$
$$= \frac{2y}{2xy}$$
$$= \frac{1}{x}$$

$u(x) = e^{\int \frac{1}{x}\,dx}$

$u(x) = e^{\ln x}$

$\boxed{u(x) = x}$

(2) Solve the equation.(4pt)

$x\left(2y^2 + 4x^2 + 2xy\frac{dy}{dx}\right) = 0$

$2xy^2 + 4x^3 + 2x^2y\frac{dy}{dx} = 0$

$P(x,y)$ $\quad$ $Q(x,y)$
$(2xy^2 + 4x^3)\,dx + 2x^2y\,dy = 0$

$\frac{\partial P}{\partial y} = 4xy$ ✓

$\frac{\partial Q}{\partial x} = 4xy$ ✓

$F(x,y) = \int P(x,y)\,dx + \phi(y)$

$= x^2y^2 + x^4 + \phi(y)$

$Q(x,y) = \frac{\partial F}{\partial y} = 2x^2y + \phi'(y)$

$2x^2y + \phi'(y) = 2x^2y$

$\therefore \phi'(y) = 0$

$\phi(y) = 0$

$F(x,y) = x^2y^2 + x^4$

$\boxed{x^2y^2 + x^4 = C}$

$xy^2 = C - x^4$

$y = \pm\sqrt{\frac{C}{x} - x^3}$

Exercise 2. (12pt) Consider the differential equation

$$\frac{dy}{dx} = \frac{y^2 - y}{x}$$

(1) Find the explicit general solution. (5pt)

$$\int \frac{1}{y^2-y}\,dy = \int \frac{1}{x}\,dx$$

$$\frac{1}{y(y-1)}$$

$$\frac{A}{y} + \frac{B}{y-1}$$

$$A(y-1) + By = 1$$

$$-A = 1$$

$$A = -1$$

$$-1 + B = 0$$

$$B = 1$$

$$\frac{-1}{y} + \frac{1}{y-1}$$

$$\int \left(-\frac{1}{y} + \frac{1}{y-1}\right) dy = \ln|x| + C_0$$

$$-\ln|y| + \ln|y-1| = \ln|x| + C_0$$

$$\ln\left|\frac{y-1}{y}\right| = \ln|x| + C_0$$

$$\left|\frac{y-1}{y}\right| = e^{\ln|x| + C_0}$$

$$\left|1 - \frac{1}{y}\right| = e^{C_0}|x|$$

$$C_1 = \pm e^{C_0}$$

$$1 - \frac{1}{y} = \pm e^{C_0} x$$

$$1 - \frac{1}{y} = C_1 x$$

$$\frac{1}{y} = 1 - C_1 x$$

$$\boxed{y = \frac{1}{1 - C_1 x}}$$

(2) Find the solution to this equation that satisfies the initial condition $y(1) = 2$. (2pt)

$$2 = \frac{1}{1 - C_1}$$

$$2(1 - C_1) = 1$$

$$2 - 2C_1 = 1$$

$$-2C_1 = -1$$

$$C_1 = \frac{1}{2}$$

$$y = \frac{1}{1 - \frac{1}{2}x}$$

$$\boxed{y = \frac{2}{2 - x}}$$

(3) What is the interval of existence of the solution you found in (b). (3pt)

$$y = \frac{2}{2-x}$$

$2 - x \neq 0$

$x \neq 2$ since initial value of $1 < 2$, only consider $(-\infty, 2)$

$\boxed{(-\infty, 2)}$

(4) Find the solution to this equation that satisfies the initial condition $y(1) = 0$. (2pt)

$$y = \frac{1}{1 - c_1 x}$$

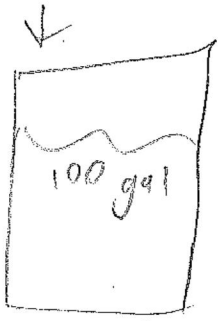
$$0 = \frac{1}{1-1}$$

$$0 = \frac{1}{0} \times$$

∴ only solution is

$\boxed{y(x) = 0.}$

Exercise 3. (7pt) Suppose there is a tank filled with 100 gallons of water. Pure acid flows into the tank at a rate of 4 gal/min and the well mixed solution leaves the tank at the of 2 gal/min rate. Let $x(t)$ be the volume in gallons of acid in the tank at time t. Find $x(t)$ for any given time t.



4 gal/min

2 gal/min

rate in = 4 gal/min

rate out = $\frac{x(t)}{100+2t}$ · 2 gal/

$\frac{dx}{dt}$ = rate in − rate out

$\frac{dx}{dt} = 4 - \frac{2x(t)}{100+2t}$

$\frac{dx}{dt} = 4 - \frac{x}{50+t}$

$x' + \frac{x}{50+t} = 4$

$e^{-\int -\frac{1}{50+t}\,dt}$

$e^{\int \frac{1}{50+t}\,dt}$

$e^{\ln|50+t|}$

$50+t$

$((50+t)x)' = (50+t)(4)$

$\int((50+t)x)' = \int 200+4t$

$(50+t)x = 200t + 2t^2 + C$

$(50+t)x = 150t + 2t^2 + C$

$x = \frac{150t+2t^2}{50+t} + \frac{C}{50+t}$

$x(0) = \frac{0+0}{50+0} + \frac{C}{50}$

$0 = 0 + C$

$C = 0$

$x(t) = \frac{150t+2t^2}{50+t}$

$P(x,y) = 4yx$

$Q(x,y) = 5x^2$

Exercise 4. (7pt) Consider

$$4yxdx + 5x^2dy$$

(1) Show that the above equation is not exact. (3pt)

$\frac{\partial P}{\partial y} = 4x$ $\quad$ $\frac{\partial Q}{\partial x} = 10x$

$4x \neq 10x$

Since $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$, the equation is not exact

(2) Find a and b such that $x^a y^b$ is an integration factor of the above equation. (4pt)

$4x^{a+1}y^{b+1} + 5x^{a+2}y^b\,dy$

$\frac{\partial P}{\partial y} = 4(b+1)x^{a+1}y^b$ $\quad$ $\frac{\partial Q}{\partial x} = 5(a+2)x^{a+1}y^b$

$4(b+1)x^{a+1}y^b = 5(a+2)x^{a+1}y^b$

$4(b+1) = 5(a+2)$

$4b+4 = 5a+10$

$4b-6 = 5a$

$a = \frac{4b-6}{5}$

Let $a = 2$

$2 = \frac{4b-6}{5}$

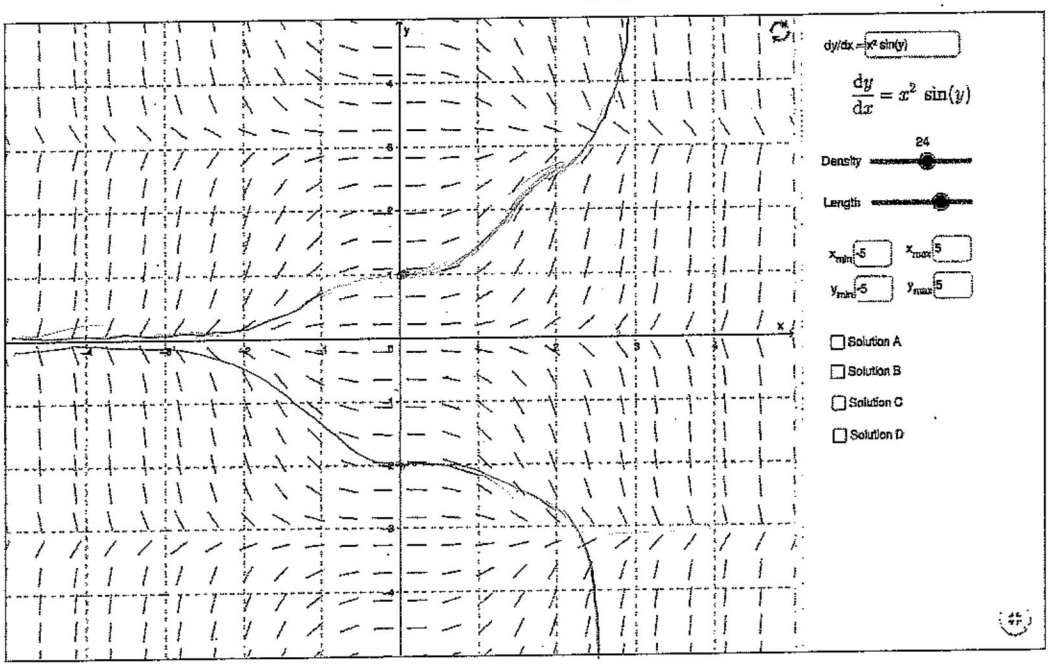
$10 = 4b-6$

$16 = 4b$

$b = 4$

$a = 2,\ b = 4$

Field M1 F18.png



1. Short answer problems

(no explanation needed)

(1) (4pt) Consider the above direction field and draw the solution through (0,1) and the solution through (0,-2).

(2) (2pt) Which of the following are homogeneous differential equations?

$\sin(x/y)\frac{dy}{dx} + 2 = 0$

$\frac{dy}{dx} = \frac{-2}{\sin(x/y)}$

(Y) / N $\sin(\frac{x}{y})dy + 2dx = 0$

Y / (N) $(xy + x^2)dy + (y^2x - x^2y)dx$ $\quad (xy+x^2)\frac{dy}{dx} + (y^2x - x^2y) = 0$

(Y) / N $\sin(xy)dy - \cos(xy)dx$

Y / (N) $\sqrt{x^2y^2 - 4xy^3}dy + x^2dx$ $\quad \frac{y^2x + yx^2}{xy - yx^2}y$