I

$y^2+y=2 \quad y \qquad y^2=-y+2$

$y_h = e^{-\int dt} = e^{-t}dt \qquad y_h = e^{\int -dt} = e^{-t}dt$

$v' = \frac{2}{e^{t}}$

MATH 33B - 1
Differential Equations – Midterm 1
Version 1B

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October 18, 2013

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TA Name: Top
Section: 1A 1B 1C 1D 1E 1F
Sect. day: Tues Thurs

Directions: This test is to be completed **without** the use of notes, books, or technology. The time limit is 50 minutes. You must show all your work to get full credit, except for the multiple choice questions (Q1 and Q2).

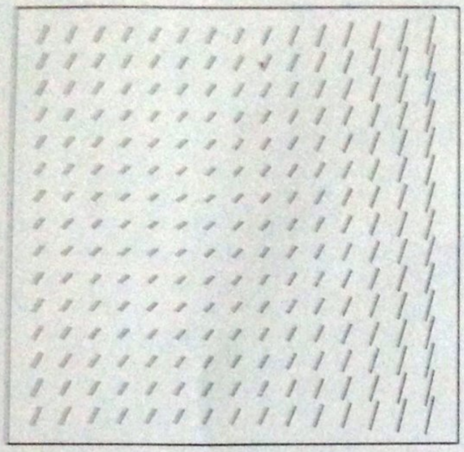
(For grading purposes only – Leave blank)

Q1	10	10
Q2	10	6
Q3	10	6
Q4	10	10
Q5	12	12
Q6	6	6
Total	58	50

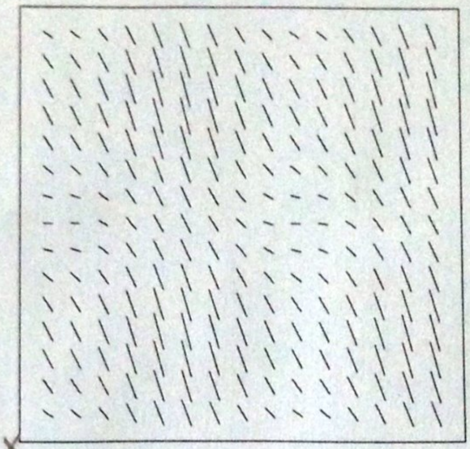
✓

Question 1. (10 pts) Please match the following slope fields with their associated differential equations. Circle the correct answers.

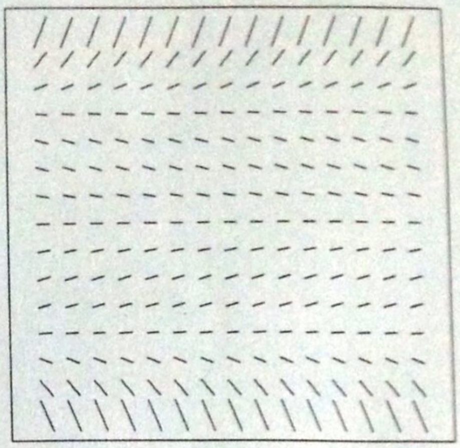
y



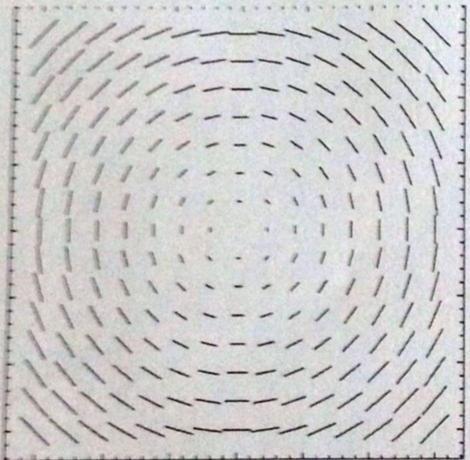
(a)



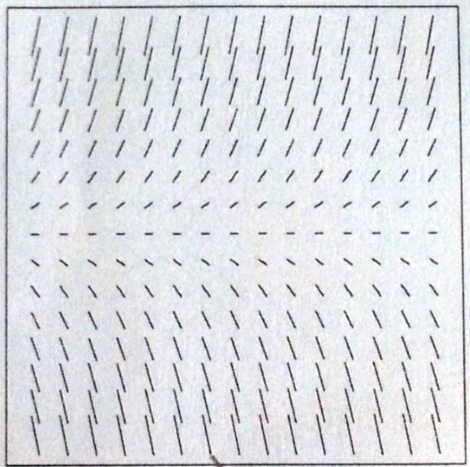
(b)



(c)



(d)



(e)

$f(y) = \frac{-x}{y}$

| x | y |

1) $y' = \frac{-x}{y}$ a b c d e

2) $y' = y(y-3)(y+3)$ a b c d e

3) $y' = y^2 + x^2 + e^x$ a b c d e

4) $y' = \sin x + \cos y - 2$ a b c d e

5) $y' = y$ a b c d e

$\frac{dy}{dx} = \frac{-x}{y}$

$y\,dy = -x$

Question 2. (10 pts) Which of the following differential 1-forms ω are exact on the domain $\mathbb{R}^2 \setminus (0,0)$? Circle the correct answer.

a)	$\omega := x\,dx + (y-x)\,dy$	Exact	Not-exact
b)	$\omega := (x+y)\,dx + (x-y)\,dy$	Exact	Not-exact
c)	$\omega := \frac{y}{x^2+y^2}dx + \frac{-x}{x^2+y^2}dy$	Exact	Not-exact
d)	$\omega := dF$, for $F(x,y) = 2x^2 + y^2$	Exact	Not-exact
e)	$\omega := dF$, for $F(x,y) = \frac{x-y}{x^2+y^2}$	Exact	Not-exact

a) $\frac{\partial P}{\partial y} = 0 \quad \frac{\partial Q}{\partial x} = -1.$

b) $\frac{\partial P}{\partial y} = 1 \quad \frac{\partial Q}{\partial x} = 1$

c) $\frac{\partial P}{\partial y} = \frac{-y\,2y}{(x^2+y^2)^2} \quad \frac{\partial Q}{\partial x} = \frac{x(2x)}{(x^2+y^2)^2}$

d) $\frac{\partial F}{\partial x} = 4x \quad \frac{\partial F}{\partial y} = 2y \quad dF = 4x\,dx + 2y\,dy$

e) $\frac{\partial F}{\partial x} = \frac{(x^2+y^2)-(x-y)2x}{(x^2+y^2)^2} \qquad \frac{\partial F}{\partial y} = \frac{-(x^2+y^2)-(x-y)2y}{(x^2+y^2)^2}$

$= \frac{x^2+y^2-2x^2+2xy}{(x^2+y^2)^2} \qquad = \frac{-x^2-y^2-2xy+2y^2}{(x^2+y^2)^2}$

$= \frac{-x^2+y^2+2xy}{(x^2+y^2)^2}\,dx \qquad = \frac{-x^2+y^2-2xy}{(x^2+y^2)^2}\,dy$

$\frac{\partial P}{\partial y} = \frac{(2y-2x)(x^2+y^2)^2 - (-x^2+y^2+2xy)\,2(x^2+y^2)\,2y}{(x^2+y^2)^4}$

6

$\frac{\partial Q}{\partial x} = \frac{(-2x-2y)(x^2+y^2)^2 - (-x^2+y^2-2xy)\,2(x^2+y^2)(2x}{(x^2+y^2)^4}$

Question 3. Consider the initial value problem

$$y' = \sin y, \qquad y(0) = 3.$$

Remember: when using any theorem from class, make sure to explain why it applies.

a) (2 pts) Does there exist a solution $y(t)$ to this initial value problem? Why?

$y' = \sin y = F(y)$

$\sin y$ is continuous everywhere on the plane $\in \mathbb{R}$

Therefore, there exists a solution $y(t)$ to the IVP.

+2

b) (2 pts) If so, how many solutions exist? Why?

$F'(y) = \cos y$ is continuous everywhere on the plane as well as $F(y)$. therefore, there exists one and only unique solution on the plane

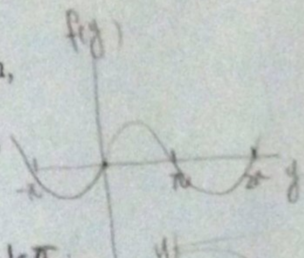
+2

(4

c) (4 pts) If $y(t)$ is a solution to the initial value problem,

$$y' = \sin y, \quad y(0) = 3,$$

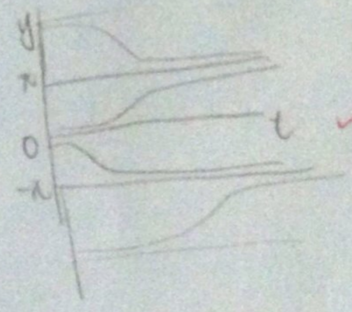
what is $\lim_{t\to\infty} y(t)$? Justify your response.



according to the graph, as $t \to \infty$, $y \to k\pi$

therefore $y \to 0$

$\lim_{t\to\infty} y(t) = \cancel{0}$



+2

d) (4 pts) If $y(t)$ is a solution to the above initial value problem, then for which value(s) of t does $y(t) = -1$? Justify your answer.

$\frac{dy}{\sin y} = dt$ $\quad \frac{1}{\sqrt{y^2-1^2}} = t + C$

2

0

$\frac{1}{\sqrt{3^2-1^2}} = C$

$\frac{1}{\sqrt{8}} = C$

$\frac{1}{\sqrt{(-1)^2-1^2}} = t + C$ doesn't exist

0

(2

Question 4. (10 pts) Solve the initial value problem:

$$y' = y + 2xe^{2x}, \quad y(0) = 3.$$

$$y' - y = 2xe^{2x}$$

$$\mu = e^{-\int dx} = e^{-x}$$

$$\mu f = \int e^{-x} \cdot 2xe^{2x}\,dx$$

$$= 2\int x e^{x} dx \qquad \begin{matrix} u = x \\ du = dx \\ v = e^{x} \\ dv = e^{x} du \end{matrix}$$

$$= 2(xe^{x} - e^{x} + C) = 2(x-1)e^{x} + C$$

$$f = \frac{2(x-1)e^{x} + C}{e^{-x}} = 2(x-1)e^{2x} + Ce^{x}$$

$$3 = 2(-1)e^{0} + Ce^{0}$$
$$= -2 + C \qquad C = 5$$

$$\therefore y(x) = 2(x-1)e^{2x} + 5e^{x}$$

10

Question 5. (12 pts) Solve the following initial value problem for an equation of the form $F(x,y) = C$ that defines y *implicitly* as a function of x:

x cos xt

$$y' = \frac{(2x\cos y + 3x^2 y)}{(y + x^2 \sin y - x^3)}, \quad y(0) = 2.$$

$$\frac{dy}{dx} = \frac{2x\cos y + 3x^2y}{y + x^2\sin y - x^3} \qquad (2x\cos y + 3x^2y)dx - (y + x^2\sin y - x^3)dy = 0$$

$$P = 2x\cos y + 3x^2y \qquad \frac{\partial P}{\partial y} = -2x\sin y + 3x^2$$

$$Q = x^3 - x^2\sin y - y \qquad \frac{\partial Q}{\partial x} = 3x^2 - 2x\sin y$$

] exact

$$f = \int 2x\cos y + 3x^2y\,dx + \phi(y) = x^2\cos y + x^3y + \phi(y)$$

$$\frac{\partial f}{\partial y} = -x^2\sin y + x^3 + \phi'(y) \qquad \phi'(y) = -y \qquad \phi(y) = -\frac{1}{2}y^2$$

$$\therefore f(x,y) = x^2\cos y + x^3y - \frac{1}{2}y^2 = C$$

$$0^2\cos 2 + 0^3\cdot 2 - \frac{1}{2}4 = C \qquad C = -2$$

$$F(x,y) = x^2\cos y + x^3y - \frac{1}{2}y^2 = -2$$

+12

Question 6. (6 pts) The differential form

$$\omega := 2ydx + (x+y)dy$$

is not exact. Find an integration factor of the form $\mu(x,y) = \mu(y)$ so that the product $\mu(y) \cdot \omega$ is equal to dF for some continuously differentiable function $F(x,y)$ on the domain $(0,\infty) \times (0,\infty)$. Note: you **do not** need to find this function F, just find $\mu(y)$.

$$h = \frac{1}{2y}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = \frac{1}{2y}(2-1) = \frac{1}{2y}$$

$$\mu(y) = e^{\int \frac{1}{2y}} = e^{\frac{1}{2}\ln y} = y^{\frac{1}{2}}$$