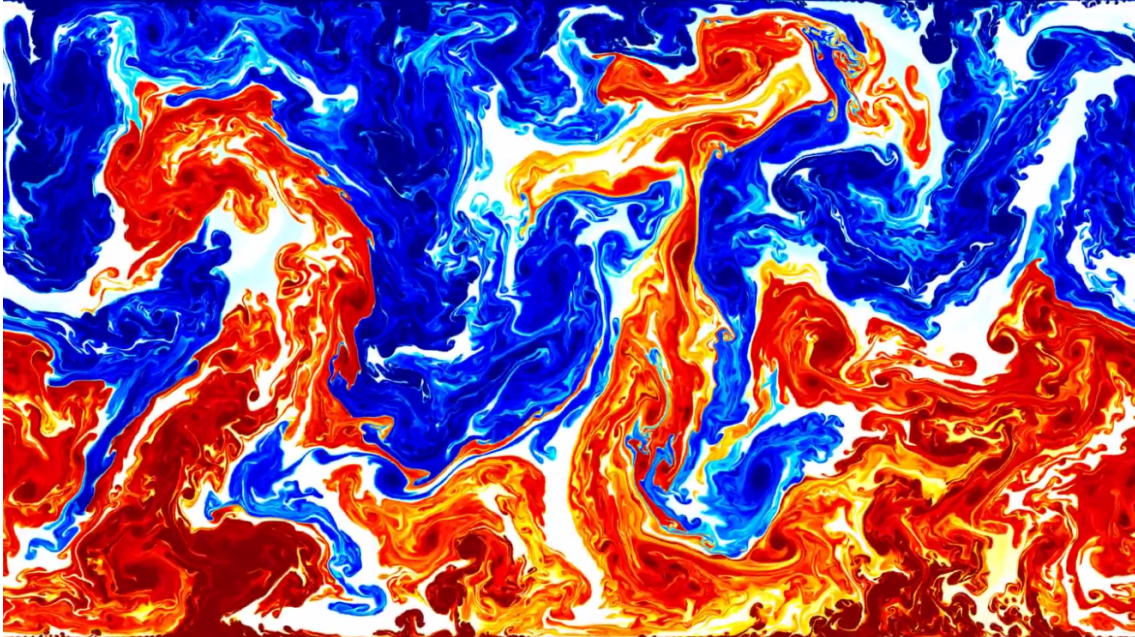


# Midterm 01

Math 33b *UCLA* (Summer 2021)

Assigned: August 25, 2021.



## Instructions/Admonishment

1. **SHOW ALL WORK**
2. Duration: 24 Hours.
3. The following is my own work, without the aid of electronic calculating devices or any other person.  
Signature: \_\_\_\_\_

**Problem 1** *Solution to Linear ODEs* .

Find the particular solution of the equation  $y'' + 4y' + 3y = 56 \sin 2x$  by

- (i) the method of undetermined coefficients,
- (ii) the method of variation of parameters (**feel free to use the formula or to show some key steps of the calculation process**)

**Problem 2** *Nonhomogeneous Equation.*

Consider the equation  $y'' - 9y = \cos x$ .

- (i) If  $y_p = -\frac{1}{10} \cos x$  is particular solution, what is the particular solution of the equation  $y'' - 9y = 12 \cos x$ ? (do not use the UC or variation of parameters method to justify your answer).
- (ii) Using (i) find the general solution of the equation  $y'' - 9y = 12 \cos x$ .

**Problem 3** *Autonomous equation* .

Consider  $\frac{dy}{dt} = e^y - 2$ , with  $-\infty < y < +\infty$ . This equation has one stable equilibrium point. Select one of the the following answers and **justify your response**.

- (i) the stable equilibrium point being  $y = \ln 2$
- (ii) the stable equilibrium point being  $y = 0$
- (iii) the equilibrium point is unstable.
- (iv) there are no equilibrium points.

**Problem 4** *Mass/Spring ODE* .

In an experiment, a  $2\text{-Kg}$  mass is suspended from a spring. The displacement of the spring-mass equilibrium from the spring equilibrium is measured to be  $50\text{ cm}$ . The mass is then displaced  $12\text{ cm}$  downward from its spring-mass equilibrium and released from rest.

- (i) Sketch the setup of the experiment showing the positive direction of the orientation of the displacement.
- (ii) Set up (but do not solve) the initial value problem that models this experiment. Assume no damping is present.