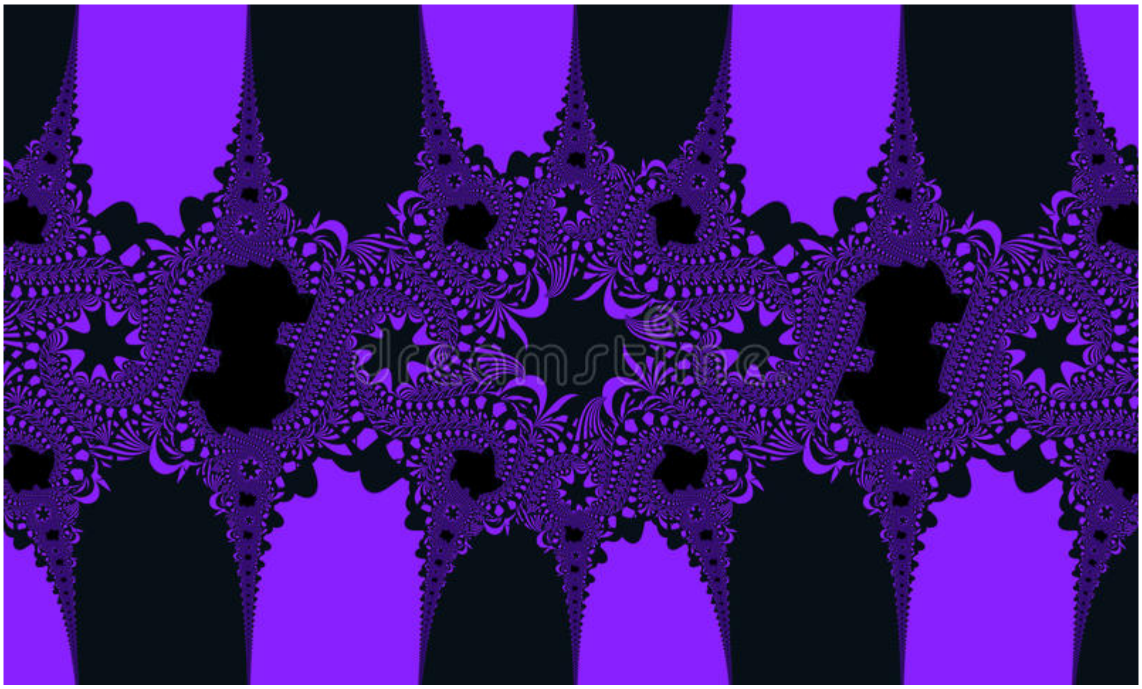
FINAL EXAM

Differential Equations @ $\mathscr{U}CLA$ (Summer 2021)

Assigned: September 08, 2021.



Instructions/Admonishment

1. **SHOW ALL WORK**
2. Duration: 24 Hours.
3. The following is my own work, without the aid of electronic calculating devices or any other person.
   Signature: ____

Problem 1 ***First order differential equation*** .

Assume that y is a function of x and solve the initial value problem.

(i) $(\sqrt{x+y}-\sqrt{x-y})\,dx+(\sqrt{x-y}-\sqrt{x+y})\,dy=0$, $y(0)=2$.

(ii) $8\cos^2 y dx+\csc^2 x dy=0$, $y(\frac{\pi}{12})=\frac{\pi}{4}$.

Problem 2 ***Nonhomogeneous equation.***

Find the solution of the differential equation with the designated method for the particular solution.

(i) $\mathscr{D}: y'' + 2y' + y = e^x \ln x$ for $x > 0$ (**use variation of parameters method**)

(ii) $\mathscr{I}: y'' + \lambda^2 y = \cos(\lambda x), y(0) = 0, y'(0) = 0$ (**use the UC method**)

Problem 3 ***Newton's law for cooling*** .

In the investigation of homicide, the time of death is important. The normal body temperature of most healthy people is $98.6°F$. Suppose that when a body is discovered at noon, its temperature is $82°F$. Two hours later it is $72°F$.

(i) Use Newton's law for cooling to solve for $T(t)$, the temperature of the body at any time t.

(ii) If the temperature of the surroundings is $65°F$, what was the approximate time of death?

Problem 4 ***Exact differential equation*** .

Consider the differential equation $\mathscr{E} : \big(1 + y^2\cos(xy)\big)\,dx + (xy\cos(xy) + \sin(xy))\,dy = 0$.

(i) Show the differential equation $\mathscr{E}$ is exact.

(ii) Solve the differential equation $\mathscr{E}$.

Problem 5 ***Linear systems*** .

Consider the system of differential equations

$$(\mathscr{S}_1) : \begin{cases} \dfrac{dx}{dt} = 6x - 17y \\[2ex] \dfrac{dy}{dt} = 8x - 6y \end{cases}$$

(i) Find the real-valued general solutions by eigenvalues/eigenvectors.

(ii) Find the solution with the initial condition $(x(0), y(0)) = (4, 2)$

Problem 6 ***Parametric linear systems .***

Consider the parametrized linear system

$$(\mathscr{S}_\alpha): \begin{cases} \dfrac{dx}{dt} = (4+\alpha)x - (8+\alpha)\,y \\ \\ \dfrac{dy}{dt} = 3x - 4y \end{cases}$$

(i) Find the eigenvalues of the coefficient matrix of the system in term of α.

(ii) State the type of equilibrium point at the origin for $\alpha = -2, 2, 6, 10$.

(iii) Sketch the phase plane for $\alpha = 10$.