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MIDTERM 2

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Math33B
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section: 2C - Wilkinson

Problem	Points	Score
1	11	
2	8	
3	6	
4	5	
5	7	
6	3	
Total	40	

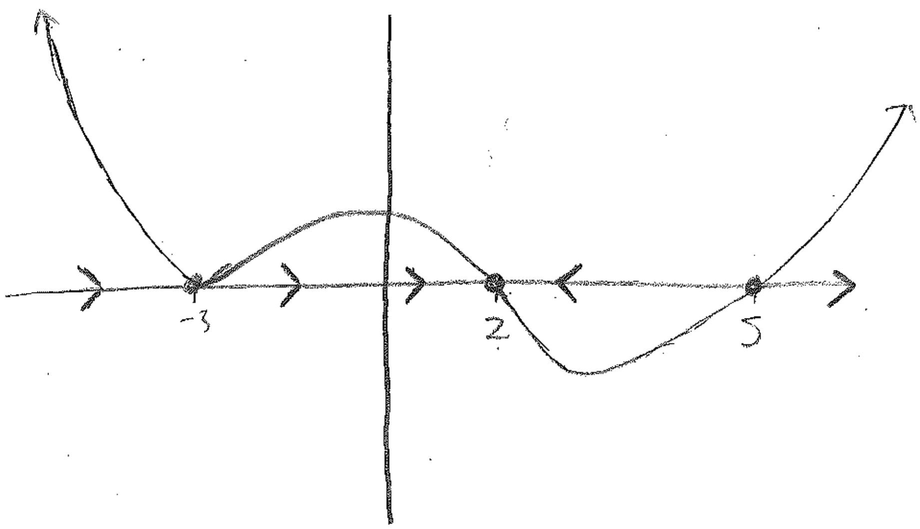
Instructions

(1) This exam has 6 problems. Make sure you have all pages.
(2) Enter your name, SID number, and discussion section on the top of this page.
(3) **Use a PEN** to record your final answers.
(4) If you need **more space**, use the extra page at the end of the exam.
(5) NO Calculators, computers, books or notes of any kind are allowed.
(6) Show your work. Unsupported answers will not receive full credit.
(7) Good Luck!

Exercise 1. (11pt)
Consider the autonomous first-order differential equation.

$$y' = (y+3)^2(y-2)(y-5)$$

(1) Draw the phase line. (3pt)



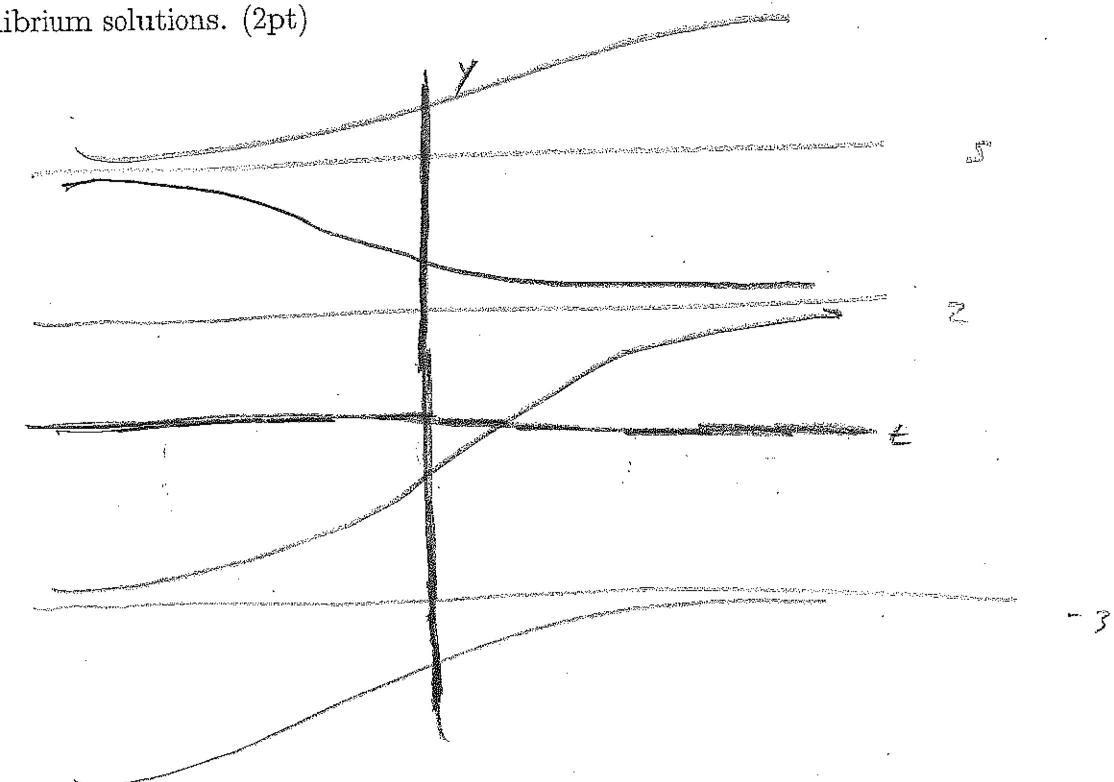
(2) What are the equilibrium solutions? Which are stable, and which are unstable? (3pt)

$y(t) = -3$ – no conclusion

$y(t) = 2$ – stable

$y(t) = 5$ – unstable

(3) Sketch the graph of at least one solutions between each pair of adjacent equilibrium solutions. (2pt)



(4) Let $y_p(t)$ be a particular solution to the equation which satisfies $y_p(0) = 0$. Is it possible that $y_p(2) = 2$? Justify your answer. (3pt)

$y' = f(t, y) = (y+3)^2(y-2)(y-5)$ – continuous $(-\infty, \infty)$

$\frac{\partial f}{\partial y}$ – also continuous $(-\infty, \infty)$

– The uniqueness theorem can be applied here since $f(t,y)$ and $\frac{\partial f}{\partial y}$ are both continuous for $(-\infty, \infty)$

– If $y_p(2) = 2$, then the point $(2, 2)$ would have two particular solutions, y_p and $y = 2$

– This would contradict the uniqueness theorem thus its impossible for $y_p(2) = 2$ if $y_p(0) = 0$

$\frac{1}{t^2}(x^2-4)^{1/2}$

Exercise 2. (8pt) Consider the differential equation

$$\frac{dx}{dt} = \frac{\sqrt{x^2-4}}{t^2}$$

(1) Can you apply the existence and uniqueness theorem to the initial value problem $x_0(1) = 6$? Justify your answer and give the biggest rectangle in which you can apply it containing the given point (if it exists). (5 pt)

$$X' = \frac{\sqrt{x^2-4}}{t^2} = f(t,x) \qquad t \neq 0$$

$$\frac{\delta F}{\delta x} = \frac{1}{t^2}\left(\frac{1}{2}\cdot 2x\,(x^2-4)^{-1/2}\right)$$

$$= \frac{x}{t^2\sqrt{x^2-4}} \qquad t \neq 0 \qquad \cancel{x \neq 2} \quad x > 2 \quad x < -2$$

- Both f and $\frac{\delta F}{\delta x}$ are continuous as long as $t \neq 0$ and $X > 2$ or $X < -2$

- ~~Existence/Uniqueness can be applied for any rectangle not containing $t=0$, $x>2$~~

- ~~Biggest Rectangle Existence/Uniqueness can be applied~~

~~$R = (0, \infty) \times (-2, 2)$~~

- ~~Existence and Uniqueness can be applied to any point in the rectangle, so the theorem can be applied to $X_0(1) = 6$~~

- Work and Answer on Extra Page

(2) Can $x_0(2) = 5$ ($x_0(t)$ is the solution to the initial value problem in part 1))?(1pt) Justify your answer. (2pt)

Equilibrium ~~conditions~~ boundaries

$X_0 = 2$

$X_0 = -2$

~~Yes~~

~~Nothing~~

No. X' is always positive for $t > 0$ and $X > 0$. Since $X_0(1) = 6$, X' would have to be negative at some point for $X_0(2) = 5$, and it is impossible for X' to be negative, so $X_0(2)$ cannot equal 5

Exercise 3. (6pt) Find a **particular** solution to the following differential equation

$$3y'' + 2y' - y = -4e^{3t}.$$

Guess: $y = Ae^{3t}$

$y' = 3Ae^{3t}$

$y'' = 9Ae^{3t}$

$27Ae^{3t} + 6Ae^{3t} - Ae^{3t} = -4e^{3t}$

$32Ae^{3t} = -4e^{3t}$

$A = -1/8$

$y = -1/8\, Ae^{3t}$

Exercise 4. (5pt) Consider the following problem:

$$y'' + y = 0 \qquad y(0) = 0 \;\; y'(\pi/2) = 0$$

(1) Show that $y(t) = C \cdot \sin(t)$ is a solution for any constant C. (3pt)

$y'(t) = C\cos(t)$

$y''(t) = -C\sin(t)$

$y'' + y = 0$

$-C\sin(t) + C\sin(t) = 0$

$0 = 0$

$\therefore y(t) = C\sin(t)$ is a solution for any constant C

(2) Why does this not violate the 2. order existence and uniqueness theorem? (2pt)

- It does not violate the 2^{nd} order existence and uniqueness theorem because the initial condition must also be applied to y', not just y

Exercise 5. (8pt) Consider the differential equation

$$y'' + \frac{1+x}{x}y' - \frac{1}{x}y = 0$$

(1) Check that $1+x$ and $\frac{2x^2+6x+4}{x+2}$ are solutions to the above equation. (4pt)

$y(x) = 1 + x$

$y'(x) = 1$

$y''(x) = 0$

$0 + \frac{1+x}{x} - \frac{1}{x}(1+x) = 0$

$0 = 0$

$\therefore$ is a solution

~~y2x~~

$y(x) = \frac{2x^2+6x+4}{x+2} = \frac{2(x^2+3x+\cancel{4}2)}{(x+2)} = \frac{2\cancel{(x+2)}(x+1)}{\cancel{(x+2)}}$

$y(x) = 2x + 2$

$y'(x) = 2$

$y''(x) = 0$

$0 + \frac{2(1+x)}{x} - \frac{2(1+x)}{x} = 0$

$0 = 0$

$\therefore$ is a solution

(2) Do they form a fundamental set of solutions?(1pt) Justify your answer. (2pt)

No. $W = \begin{pmatrix} 1+x & 2+2x \\ 1 & 2 \end{pmatrix}$

$$\det \begin{pmatrix} 1+x & 2+2x \\ 1 & 2 \end{pmatrix} = 2+2x-2-2x$$

$$= 0$$

$\therefore$ they are linearly dependent and not a fundamental set of equations

Exercise 6. (3pt)
Consider the second order equation

$$y'' - 2e^t y' - \tan(t)y = \sqrt{t^2+1}.$$

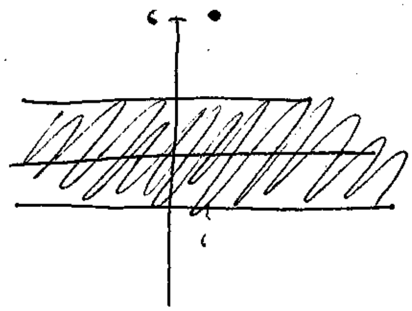
Write this equations as a planar system of first-order equations.

$\boxed{V = y'}$

$v' - 2e^t v - \tan(t)y = \sqrt{t^2+1}$

$\boxed{v' = 2e^t v + \tan(t)y + \sqrt{t^2+1}}$

2



bad area

$t \neq 0$

$x > 2$ or $x < -2$

- Both f and $\frac{\delta f}{\delta x}$ are continuous as long as $t \neq 0$ and $(x > 2$ or $x < -2)$
- Existence/Uniqueness can be applied to $x_0(1) = 6$ because there is no problems with continuity until $t = 0$ or $-2 < x < 2$
- The largest rectangle that can be used is

$$\boxed{R: \underbrace{(0, \infty)}_{t} \times \underbrace{(2, \infty)}_{x}}$$

Extra page