Midterm 1

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Math33B
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section: 2C – Wilkinson

Problem	Points	Score
1	8	
2	12	
3	7	
4	7	
SA	6	
Total	40	

Exercise 1. (8pt)
Consider the differential equations

$$2y^2 + 4x^2 + 2xy\frac{dy}{dx} = 0$$

(1) Find the integrating factor for the above equations.(4pt)
(Hint: it only depends on x)

$$(2y^2 + 4x^2)\,dx + (2xy)\,dy = 0$$

$$h = \frac{1}{Q}\left(\frac{\delta P}{\delta y} - \frac{\delta Q}{\delta x}\right) = \frac{1}{2xy}\left(4y - 2y\right) = \frac{2y}{2xy} = \frac{1}{x}$$

$$\mu(x) = e^{\int h(x)\,dx} = e^{\int \frac{1}{x}dx} = e^{\ln|x|} = |x|$$

(2) Solve the equation.(4pt)

$$x^2|x| = |x^3|$$

$$(2y^2|x| + 4x^2|x|)\,dx + (2xy|x|)\,dy = 0$$

$$F(x,y) = \int 2y^2|x| + 4x^2|x|\,dx + \Phi(y)$$

$$= 2y^2\left(\frac{x|x|}{2}\right) + 4\int x^2|x|\,dx + \Phi(y)$$

$$+ 4\int |x^3|\,dx$$

$$= y^2x|x| + |x^4| + \Phi(y)$$

$$= y^2x|x| + x^4 + \Phi(y)$$

$$\frac{\delta F}{\delta y} = 2yx|x| + \Phi'(y) = 2yx|x|$$

$$\Phi'(y) = 0$$

$$\Phi(y) = 0$$

$$F(x,y) = y^2x|x| + x^4 = C$$

Exercise 2. (12pt) Consider the differential equation

$$\frac{dy}{dx} = \frac{y^2 - y}{x}$$

(1) Find the explicit general solution. (5pt)

$$\frac{1}{y^2 - y}\,dy = \frac{1}{x}\,dx \qquad \int \frac{1}{u^2+u}$$

$$\int \frac{1}{y(y-1)}\,dy = \int \frac{1}{x}\,dx \qquad \int \frac{1}{y(y-1)}\,dy$$

$$u = \frac{1}{y-1} \qquad dv = \frac{1}{y}\,dy$$

$$du = -1(y-1)^{-2} \qquad v = \ln(y)$$

assume $y \neq 0$, $y \neq 1$

$$\longrightarrow \int \frac{1}{y-1}\,dy - \int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\ln|y-1| - \ln|y| = \ln|x| + C$$

assume $y \neq 0$ $\longrightarrow$

$$\ln \frac{|y-1|}{|y|} = \ln|x| + C$$

$$\frac{|y-1|}{|y|} = e^{C} e^{\ln|x|} \qquad \frac{|y-1|}{|y|} = e^{C_1}|x|$$

$$\boxed{\frac{|y-1|}{|y|} = C_2|x|}$$

(2) Find the solution to this equation that satisfies the initial condition $y(1) = 2$. (2pt)

$$\frac{|2-1|}{|2|} = C|1| \qquad C = \frac{1}{2}$$

$$\frac{|y-1|}{|y|} = \frac{1}{2}|x|$$

(3) What is the interval of existence of the solution you found in (b). (3pt)

$(1, \infty)$

(4) Find the solution to this equation that satisfies the initial condition $y(1) = 0$. (2pt)

$y(x) = 0$

Exercise 3. (7pt) Suppose there is a tank filled with 100 gallons of water. Pure acid flows into the tank at a rate of 4 gal/min and the well mixed solution leaves the tank at the of 2 gal/min rate. Let $x(t)$ be the volume in gallons of acid in the tank at time t. Find $x(t)$ for any given time t.

$$\frac{dx}{dt} = 4 - \frac{2x}{100+2t}$$

$$x' = \frac{-1}{50+t}x + 4$$

t is always positive

$$u(t) = e^{\int \frac{1}{50+t}} = e^{\ln|50+t|} = |50+t| = 50+t$$

$$(50+t)x' + x = 200 + 4t$$

$$((50+t)x)' = 200 + 4t$$

$$(50+t)x = \int 200 + 4t \, dt$$

$$(50+t)x = 200t + 2t^2 + C$$

$$X(t) = \frac{200t + 2t^2 + C}{50+t}$$

$$X(0) = 0$$

$$0 = \frac{0 + 0 + C}{50}$$

$$C = 0$$

$$X(t) = \frac{200t + 2t^2}{50+t}$$

Exercise 4. (7pt) Consider

$$4yxdx + 5x^2dy$$

(1) Show that the above equation is not exact. (3pt)

$$\frac{\partial P}{\partial y} = 4x \qquad \frac{\partial Q}{\partial x} = 10x$$

$$4x \neq 10x$$

$$\therefore \frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$$

$\therefore$ not exact

(2) Find a and b such that $x^a y^b$ is an integration factor of the above equation. (4pt)

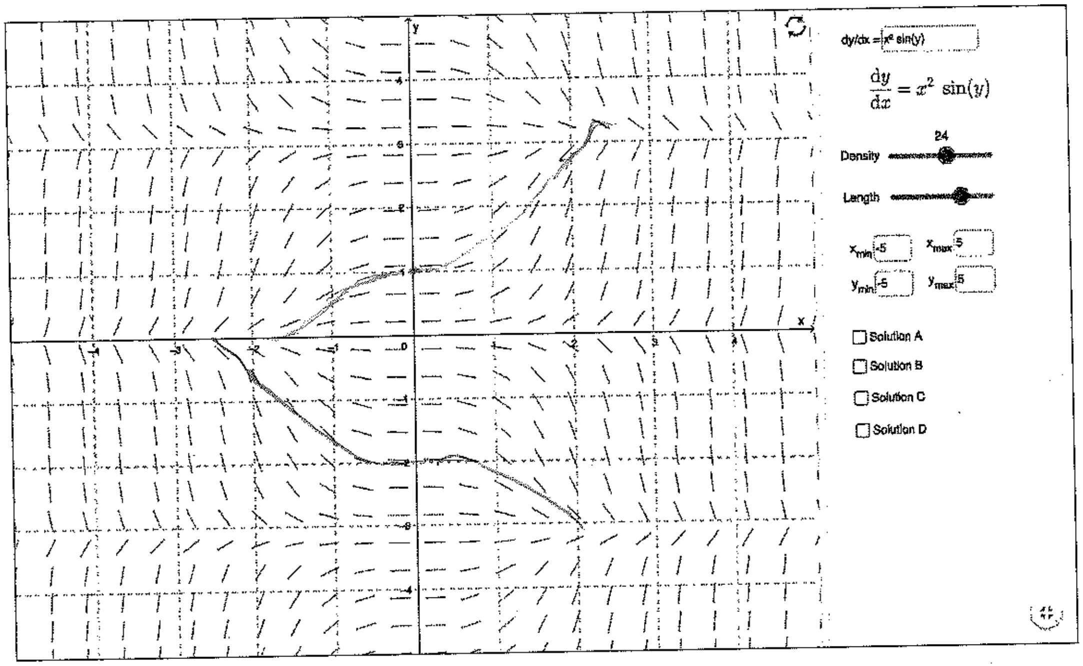
$$h = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = \frac{1}{5x^2}(-6x) = \frac{-6}{5x}$$

$$\mu(x) = e^{-\frac{6}{5}\ln x} = x^{-6/5}$$

$$a = -6/5$$

$$b = 0$$

Field M1 F18.png



1. SHORT ANSWER PROBLEMS

(no explanation needed)

(1) (4pt) Consider the above direction field and draw the solution through (0,1) and the solution through (0,-2).

(2) (2pt) Which of the following are homogeneous differential equations?

Y / N $\sin(\frac{x}{y})dy + 2dx = 0$

Y / N $(xy + x^2)dy + (y^2x - x^2y)dx$

Y / N $\sin(xy)dy - \cos(xy)dx$

Y / N $\sqrt{x^2y^2 - 4xy^3}dy + x^2dx$