20W-MATH33B-1 Midterm 2

YI SHENG TAY

TOTAL POINTS

49 / 50

QUESTION 1

Q1 10 pts

1.1 Q1a 2 / 2

✓ **- 0 pts Correct**

- **1 pts** Arithmatic mistake
- **1 pts** didn't verify solutions.
- **1 pts** didn't verify linear independence

1.2 Q1b 8 / 8

✓ **- 0 pts Correct**

- **1 pts** Minor computational mistake
- **2 pts** Forgot to put in normal form
- **2 pts** Incorrect formula for variation of parameters
- **1 pts** Did not correctly evaluate integrals
- **2 pts** Did not correctly find y after finding v's.
- **2 pts** Did not find v's.
- **4 pts** Bad guess for method of undetermined coefficients.

QUESTION 2

Q2 15 pts

2.1 Q2a 5 / 5

✓ **- 0 pts Correct**

- **2 pts** Minor mistake
- **2 pts** Solution?
- **1 pts** one solution is not correct

2.2 Q2b 9 / 10

- **0 pts** Correct
- **1 pts** minor mistake 1

✓ **- 1 pts minor mistake 2**

- **1 pts** minor mistake 3
- **2 pts** major mistake 1: exp
- **2 pts** major mistake 2: trig
- **2 pts** major mistake 3: polynomial
- **6 pts** VoP calculation not finished
- **4 pts** VoP Calc wrong
- **3 pts** y_p_1 = ?
- **7 pts** VoP No Calculation.
- **9 pts** nothing meaningful
- **8 pts** Click here to replace this description.

QUESTION 3

3 Q3 10 / 10

✓ **- 0 pts Correct**

- **2 pts** not correct eigenvalues
- **3 pts** not correct eigenvectors
- **8 pts** tried
- **1 pts** miscellineous mistake
- **1 pts** one of eigenvalue incorrect
- **1.5 pts** one of eigenvector incorrect
- **2 pts** didn't know the definition of fundamental set

QUESTION 4

Q4 15 pts

4.1 a 12 / 12

✓ **- 0 pts Correct**

- **2 pts** k is incorrect
- **5 pts** Didn't find v2 and y2
- **2 pts** Your v2 is incorrect.
- **3 pts** Found v1 and v2, but form of general solution is wrong
- **2 pts** Your v1 is incorrect
- **2 pts** Your v2 should not be an eigenvector too.
- **2 pts** Your characteristic polynomial is incorrect

4.2 b 3 / 3

✓ **- 0 pts Correct**

- **0 pts** Incorrect, but correct given answer in (a)

- **2 pts** System solved/written incorrectly
- **1 pts** Write down the actual solution, not just the coefficients.
- **1 pts** System solved correctly, solution written down incorrectly

Haofei Fan | Math 33B | Wed, Feb 19, 2020

Midterm 2

Last Name: TAY

First Name: YI SHENG

Student ID: 005-211-361

Signature:

Section:

Tuesday:	Thursday:	
1A	(1B)	TA: YIH, SAMUEL
1C	1D	TA: KIM, BOHYUN
1E	1F	TA: BOSCHERT, NICHOLAS

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators,** books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. You must **show your work** to receive credit. Please **circle or box your final answers.**

Please do not write below this line.

Question	Points	Score
1	10	
2	15	
3	10	
4	15	
Total:	50	

1. (a) (2 points) Verify that $y_1(t) = t$ and $y_2(t) = t^3$ are two linearly independent solutions to the differential equation:

$$t^2y'' - 3ty' + 3y = 0.$$

$y_1' = 1 \quad y_1'' = 0$ $\qquad y_2' = 3t^2 \quad y_2'' = 6t$

$t^2y_1'' - 3ty_1' + 3y_1 = t^2(0) - 3t(1) + 3(t)$

$= 0$

$t^2y_2'' - 3ty_2' + 3y_2 = t^2(6t) - 3t(3t^2) + 3(t^3)$

$= 6t^3 - 9t^3 + 3t^3$

$= 0$

$$W(y_1, y_2) = \begin{vmatrix} t & t^3 \\ 1 & 3t^2 \end{vmatrix} = 3t^3 - t^3 = 2t^3 \neq 0$$

$\therefore$ y_1 and y_2 are two lin. ind. solutions to the D.E.

(b) (8 points) Find a particular solution y_p to the following differential equation:

$$t^2y'' - 3ty' + 3y = t^2 + 1$$

$y_p = At^2 + Bt + C$

$y_p' = 2At + B$

$y_p'' = 2A$

$t^2(2A) - 3t(2At + B) + 3(At^2 + Bt + C)$

$= 2At^2 - 6At^2 - 3Bt + 3At^2 + 3Bt + 3C$

$= -At^2 + 3C$

$-At^2 + 3C = t^2 + 1$

$-A = 1 \qquad 3C = 1$

$A = -1 \qquad C = \frac{1}{3}$

$$\boxed{y_p = -t^2 + \frac{1}{3}}$$

Last six digits of UID: _____

2. (a) (5 points) Find the general solution to the differential equation:

$$y'' + y' - 2y = 0$$

$\lambda^2 + \lambda - 2 = 0$

$(\lambda + 2)(\lambda - 1) = 0$

$\lambda = -2, \quad \lambda = 1$

$$\boxed{y = C_1 e^{-2t} + C_2 e^{t}}$$

(b) (10 points) Find a particular solution to the differential equation (Hint: split forcing term into three parts):

$$y'' + y' - 2y = 3e^{-2t} + 10\cos t + t + 1.$$

$y_1'' + y_1' - 2y_1 = 3e^{-2t}$

$y_2'' + y_2' - 2y_2 = 10\cos t$

$y_3'' + y_3' - 2y_3 = t + 1$

$y_{p_1} = Ae^{-2t} \quad y_{p_1}' = -2Ae^{-2t} \quad y_{p_1}'' = 4Ae^{-2t}$

$y_{p_1}'' + y_{p_1}' - 2y_{p_1} = 4Ae^{-2t} - 2Ae^{-2t} - 2Ae^{-2t}$

$= 0$

$\Rightarrow$ Try $y_{p_1} = Ate^{-2t}, \quad y_{p_1}' = Ae^{-2t} - 2Ate^{-2t}, \quad y_{p_1}'' = -2Ae^{-2t} - 2Ae^{-2t} + 4Ate^{-2t}$

$y_{p_1}'' + y_{p_1}' - 2y_{p_1} = 4Ate^{-2t} - 4Ae^{-2t} + Ae^{-2t} - 2Ate^{-2t} - 2Ate^{-2t}$

$= 4Ate^{-2t} - 4Ae^{-2t}$

$= -3Ae^{-2t}$

$-3Ae^{-2t} = 3e^{-2t} \Rightarrow A = -1 \qquad \underline{y_1 = -te^{-2t}}$

$y_{p_2} = A\cos t + B\sin t, \quad y_{p_2}' = -A\sin t + B\cos t, \quad y_{p_2}'' = -A\cos t - B\sin t$

$y_{p_2}'' + y_{p_2}' - 2y_{p_2} = -A\cos t - B\sin t - A\sin t + B\cos t - 2A\cos t - 2B\sin t$

$= (-3A + B)\cos t + (-3B - A)\sin t$

$B - 3A = 10 \qquad A = 3B \qquad B = -\frac{5}{4}, \; A = -\frac{15}{4}$

$\underline{y_2 = -\frac{15}{4}\cos t - \frac{5}{4}\sin t}$

$y_{p_3} = At + B, \quad y_{p_3}' = A, \quad y_{p_3}'' = 0 \qquad -2A = 1 \quad A = -\frac{1}{2}$

$y_{p_3}'' + y_{p_3}' - 2y_{p_3} = 0 + A - 2At - 2B = -2At + A - 2B \qquad A - 2B = 1 \quad B = -\frac{3}{4}$

$\underline{y_3 = -\frac{1}{2}t - \frac{3}{4}}$

$$\boxed{y_p = y_1 + y_2 + y_3 = -te^{-2t} - \frac{15}{4}\cos t - \frac{5}{4}\sin t - \frac{1}{2}t - \frac{3}{4}}$$

3. (10 points) Find the fundamental set (in other words, you have to find two linearly indepedent solutions) of the following 2×2 system $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 4 \\ 3 & 1-\lambda \end{vmatrix}$$

$$= (2-\lambda)(1-\lambda) - 12$$

$$= 2 - \lambda - 2\lambda + \lambda^2 - 12$$

$$= \lambda^2 - 3\lambda - 10$$

$$= (\lambda - 5)(\lambda + 2)$$

$$\lambda = -2, \quad \lambda = 5$$

$\lambda = -2$,

$$\begin{pmatrix} 4 & 4 \\ 3 & 3 \end{pmatrix} v_1 = 0$$

$$v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\lambda = 5$,

$$\begin{pmatrix} -3 & 4 \\ 3 & -4 \end{pmatrix} v_2 = 0$$

$$v_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Fundamental set:

$$y_1 = C_1 e^{-2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$y_2 = C_2 e^{5t} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Last six digits of UID: _____

4. (a) (12 points) Find the general solution ($y_{\text{general}} = C_1 y_1(t) + C_2 y_2(t)$) to the following 2×2 system $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} 4 & 4 \\ -1 & 8 \end{pmatrix}.$$

$$|A - \lambda I| = \begin{vmatrix} 4-\lambda & 4 \\ -1 & 8-\lambda \end{vmatrix}$$

$$= (4-\lambda)(8-\lambda) + 4$$

$$= 32 - 8\lambda - 4\lambda + \lambda^2 + 4$$

$$= \lambda^2 - 12\lambda + 36$$

$$(\lambda - 6)^2 = 0$$

$\lambda = 6$,

$$\begin{pmatrix} -2 & 4 \\ -1 & 2 \end{pmatrix} v_1 = 0$$

$$v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 4 \\ -1 & 2 \end{pmatrix} v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$y_{\text{general}} = C_1 e^{6t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{6t} \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right]$$

(b) (3 points) Find a solution to the above differential equation with initial condition $y(0) = (1, 1)^T$. (In other words, determine coefficient C_1 and C_2. Try simplify your answers.)

$$y(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$2C_1 + C_2 = 1$$

$$C_1 + C_2 = 1$$

$$C_1 = 0, \quad C_2 = 1$$

$$y = e^{6t} \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right]$$

Scratch Paper and Some useful formulas, etc:

Variation of Parameters, (2nd Order Differential Equations)

$$v_1(x) = -\int \frac{1}{W} y_2(x) f(x) \mathrm{d}x$$

$$v_2(x) = \int \frac{1}{W} y_1(x) f(x) \mathrm{d}x$$