20W-MATH33B-1 Final Exam

THILAN TRAN

TOTAL POINTS

87 / 100

QUESTION 1

1 Question 1 10 / 10

✓ **- 0 pts Correct**

- 1 pts minus sign error

- 2 pts need to be in the form of "y = " form

- 3 pts algebraic mistake need to be ln(x)

- 1 pts miscelleneous algebraic mistake

- 5 pts only the homogeneous equation substitution was right

- 2 pts the position of "+c" needs to be in denominator

- 8 pts tried

QUESTION 2

2 Question 2 15 / 15

✓ **- 0 pts Correct**

- 3 pts some portion particular solution wrong/not found correctly

- 1 pts miscellaneous mistake

- 6 pts answer for part (b) wrong

- 2 pts general solution for 2(a) incomplete

QUESTION 3

Question 3 15 pts

3.1 3 (a) 10 / 10

✓ **- 0 pts Correct**

- 4 pts Miscomputed one eigenvector.

- 2 pts You found fundamental solutions, but forgot C_1 and C_2.

- 3 pts Miscomputed both eigenvectors.

- 1 pts Write down the general solution, not just fundamental solutions.

3.2 3(b) 3 / 5

- 0 pts Correct

- 1 pts Justification?

✓ **- 2 pts Eigenvectors graphed in incorrect quadrants.**

- 2 pts Indicate direction travelled on solution curves.

- 2 pts The shape of your curves as t goes to infinity or - infinity is wrong

- 3 pts Draw solution curves in quadrants cut out by eigenvectors.

- 3 pts Draw the half-line solutions (the ones corresponding to the eigenvectors).

QUESTION 4

4 Question 4 10 / 10

✓ **- 0 pts Correct**

- 2 pts Your third fundamental solution is wrong/missing.

- 2 pts Your second fundamental solution is wrong/missing.

- 1 pts e^{2t} not e^t.

- 2 pts System of coefficients solved incorrectly/solution to IVP missing/incorrect.

- 2 pts Your first fundamental solution is wrong/missing.

QUESTION 5

5 Question 5 13 / 15

- 0 pts Correct

- 2 pts Identify block matrices

- 3 pts Find eigenvalues for each block

- 3 pts Find (generalized) eigenvectors for each block

- 4 pts Construct solutions for each block

- 3 pts Combine solutions.

- 1 pts Minor calculation error

✓ - **2 pts** Moderate error in solution for one block

We want real valued solutions

QUESTION 6

Question 6 15 pts

6.1 6(a) 5 / 5

✓ - **0 pts** Correct

- **2 pts** Knew to find zeros of RHS.

- **2 pts** Correctly found at least one infinite family of solutions.

- **1 pts** Found half of the solutions or made a computational mistake.

6.2 6(b) 5 / 5

✓ - **0 pts** Correct

- **2 pts** Included equilibria

- **3 pts** Solutions go in correct directions

- **1 pts** Violates uniqueness

- **1 pts** Small error

6.3 6(c) 5 / 5

✓ - **0 pts** Correct

- **2 pts** Invoke hypotheses of existence and uniqueness.

- **3 pts** Bound by equilibria

- **1 pts** Minor error

- **1 pts** Show it satisfies the hypotheses (show f and f' are continuous).

- **1 pts** State that there are arbitrarily large or small equilibria.

- **1 pts** Invoke existence and uniqueness.

QUESTION 7

Question 7 10 pts

7.1 7(a) 1 / 5

- **0 pts** Correct

- **2 pts** y_h /IF correct, didn't find y_p

- **2 pts** minor mistake / gap

✓ - **4 pts** Major mistake/gap

- **5 pts** blank

- **3 pts** y_h/IF minor mistake/not simplified, didn't find y_p

- **1 pts** y_h not simplifed

- **3 pts** y_h = ?

- **1 pts** typo

7.2 7(b) 0 / 3

- **0 pts** Correct:

- **1 pts** minor mistake (e.g. forget to say C can be anything), gap, logic flow not clear

- **2 pts** some meaningful writings. not much detail provided, many gaps.

✓ - **3 pts** nothing meaningful

7.3 7(c) 0 / 2

- **0 pts** Correct

- **2 pts** wrong

✓ - **1 pts** didn't put in normal form y' = F(y,t) = 1/ (t^2 - a^2) y + ...

✓ - **1 pts** didn't check/state that \partial F/ \partial y = 1/(t^2 - a^2) or calculation is wrong

- **1 pts** Gap

QUESTION 8

8 Question 8 10 / 10

✓ - **0 pts** Correct

- **4 pts** gap: did not verify (A-aI)^2 = 0

- **4 pts** minor mistake

- **2 pts** lack essential detail / some typos

- **4 pts** based on your flow, you didn't use math induction to give a proof for A^n

- **8 pts** Major mistake

Name: Thitan Tran ID: 605140530

$\boxed{\text{FINAL}}$

Q1: Solve $(-xy+y^2)dx + x^2dy = 0$ $P(x,y)dx + Q(x,y)dy = 0$

Check exact: $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$?

$$\frac{\partial P}{\partial y} = -x+2y \neq 2x = \frac{\partial Q}{\partial x}$$

Check depending on one factor:

$$h = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) \qquad g = \frac{1}{P}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$$

$$= \frac{1}{x^2}(-x+2y-2x) \qquad = \frac{1}{-xy+y^2}(-3x+2y)$$

$$= \frac{1}{x^2}(-3x+2y) \qquad \times$$

$\times$

Check homogenous:

$x^2 \Rightarrow t^2x^2$, homogenous w/ degree 2.

$-xy+y^2 \Rightarrow -t^2xy+t^2y^2 = t^2(-xy+y^2)$, homogenous w/ degree 2.

✓ Equation is homogenous.

Substitute $y = vx$, $dy = dvx + vdx$.

$$(-vx^2+v^2x^2)dx + x^2(dvx+vdx) = 0$$

$$(\cancel{-vx^2}+v^2x^2+\cancel{vx^2})dx + x^3dv = 0$$

$$x^2(v^2dx + xdv) = 0$$

$$v^2x^2dx + xdv = 0 \leftarrow \text{separable}$$

$$\int\frac{dx}{x} + \int\frac{dv}{v^2} = 0$$

$$\ln|x| - \frac{1}{v} = C$$

$$x - \frac{1}{v} = \ln|x| + C$$

$$v = \frac{1}{\ln|x|+C}$$

$$\frac{y}{x} = \frac{1}{\ln|x|+C} \qquad \boxed{y = \frac{x}{\ln|x|+C}}$$

1 Question 1 **10 / 10**

- ✓ **- 0 pts** **Correct**
- **- 1 pts** minus sign error
- **- 2 pts** need to be in the form of "y = " form
- **- 3 pts** algebraic mistake need to be ln(x)
- **- 1 pts** miscelleneous algebraic mistake
- **- 5 pts** only the homogeneous equation substitution was right
- **- 2 pts** the position of "+c" needs to be in denominator
- **- 8 pts** tried

Name: Thilan Tran ID: 60714053O

Q2. (a)

$$y'' - 2y' + y = 0$$
$$\lambda^2 - 2\lambda + 1 = 0$$
$$(\lambda - 1)^2 = 0$$

1 repeated root:

$y_1 = e^t$, $y_2 = te^t$

$\Rightarrow \boxed{y(t) = (C_1 + C_2 t)e^t}$

Solu. are of the form $C_1 e^{\lambda t}$ & $C_2 t e^{\lambda t}$ for repeated real roots.

Q2. (b) $y'' - 2y' + y = e^t(t+1) + e^t \sin t$

Can split using linearity.

$y'' - 2y' + y = e^t(t+1)$

Try $y = e^t(at+b)$

$y' = ae^t + e^t(at+b) = ae^t + y$

$y'' = ae^t + ae^t + e^t(at+b) = ae^t + y'$

$2\cancel{a}e^t + e^t(at+b) - 2\cancel{a}e^t - 2e^t(at+b)$

$+ e^t(at+b) = e^t(t+1)$

$0 = e^t(t+1)$

Try $y = te^t(at+b)$

$= e^t(at^2 + bt)$

$y' = (2at+b)e^t + e^t(at^2+bt) = (2at+b)e^t + y$

$y'' = 2ae^t + e^t(2at+b) + (2at+b)e^t + e^t(at^2+bt) = 2ae^t + e^t(2at+b) + y'$

$2ae^t + \cancel{2e^t(2at+b)} + \cancel{e^t(at^2+bt)}$

$- \cancel{2e^t(2at+b)} - \cancel{2e^t(at^2+bt)}$

$+ \cancel{e^t(at^2+bt)} = e^t(t+1)$

$2a\cancel{e^t} = \cancel{e^t}(t+1)$

$a = \frac{t+1}{2}$

~~$y(t) = e^t$~~ ? Try variation of parameters instead

$y'' - 2y' + y = e^t \sin t$

Try $y = e^t(a\cos t + b\sin t)$.

$y' = e^t(a\cos t + b\sin t) + e^t(-a\sin t + b\cos t)$

$= y + e^t(-a\sin t + b\cos t)$

$y'' = y' + e^t(-a\cos t - b\sin t) + e^t(-a\sin t + b\cos t)$

$y'' - 2y' + y = \cancel{y'} + e^t(-a\cos t - b\sin t) + \cancel{e^t(-a\sin t + b\cos t)} - 2\cancel{y} - 2\cancel{e^t(-a\sin t + b\cos t)} + \cancel{y} = e^t \sin t$

$= + \cancel{e^t}\cancel{(-a\sin t + b\cos t)} + \cancel{e^t}(-a\cos t - b\sin t)$

$= \cancel{e^t}\sin t$.

$a = 0$, $b = -1$

$y(t) = -e^t \sin t$

$y_p(t) = -e^t \sin t$

Try variation of parameters,

$y(t) = (v_1 + v_2 t)e^t$

$v_1 = \int$

CONTINUED →

Name: Thilan Tran ID = 605140530

Try variation of parameters.

Q2.(b) (con.)

$y_1 = e^t$, $y_2 = te^t$

$y_p = v_1 y_1 + v_2 y_2$

$$v_1 = \int \frac{-y_2\, g(t)\,dt}{y_1 y_2' - y_1' y_2} = \int \frac{-te^t\,(e^t(t+1))}{e^t[te^t + e^t] - te^{t^2}}\,dt$$

$$= \int \frac{t\cancel{e^{t^2}}(-1-t)}{\cancel{e^{t^2}}(t+1-t)}\,dt = \int \frac{-t-t^2}{1}\,dt$$

$$= -1\int t^2 + t\,dt$$

$$= -\frac{t^3}{3} - \frac{t^2}{2} + C$$

Choose $v_1 = -\frac{t^3}{3} - \frac{t^2}{2}$.

$$v_2 = \int \frac{y_1\, g(t)\,dt}{y_1 y_2' - y_1' y_2}$$

$$= \int \frac{e^t(e^t(t+1))}{e^{t^2}(t+1-t)}\,dt = \int t+1\,dt$$

$$= \frac{t^2}{2} + t + C$$

$-\frac{1}{3} + \frac{1}{2}$

Choose $v_2 = \frac{t^2}{2} + t$.

$$y_p = \left(-\frac{t^3}{3} - \frac{t^2}{2}\right)e^t + \left(\frac{t^2}{2} + t\right)te^t$$

$$y_p = \left(\frac{1}{6}t + \frac{1}{2}\right)t^2 e^t$$

$$\boxed{y_p(t) = e^t\left[\left(\frac{1}{6}t + \frac{1}{2}\right)t^2 - \sin t\right]}$$

2 Question 2 **15 / 15**

- ✓ **- 0 pts** Correct
- **- 3 pts** some portion particular solution wrong/not found correctly
- **- 1 pts** miscellaneous mistake
- **- 6 pts** answer for part (b) wrong
- **- 2 pts** general solution for 2(a) incomplete

Name: Thilan Tran ID: 605140530

Q3.(a)

$$\vec{y}' = A\vec{y}, \quad A = \begin{pmatrix} -2 & -2 \\ 2 & 3 \end{pmatrix}$$

$T = -2+3 = +1$

$D = -6-(-4)$
$= -6+4 = -2$

$$\lambda^2 - T\lambda + D = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

$$(\lambda - 2)(\lambda + 1) = 0$$

$\lambda_1 = -1$:

$(A + 1I) = \begin{pmatrix} -1 & -2 \\ 2 & 4 \end{pmatrix} \cdot \vec{v}_1 = \vec{0}$

$\vec{v}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\vec{y}_1(t) = e^{-t}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\lambda_2 = 2$:

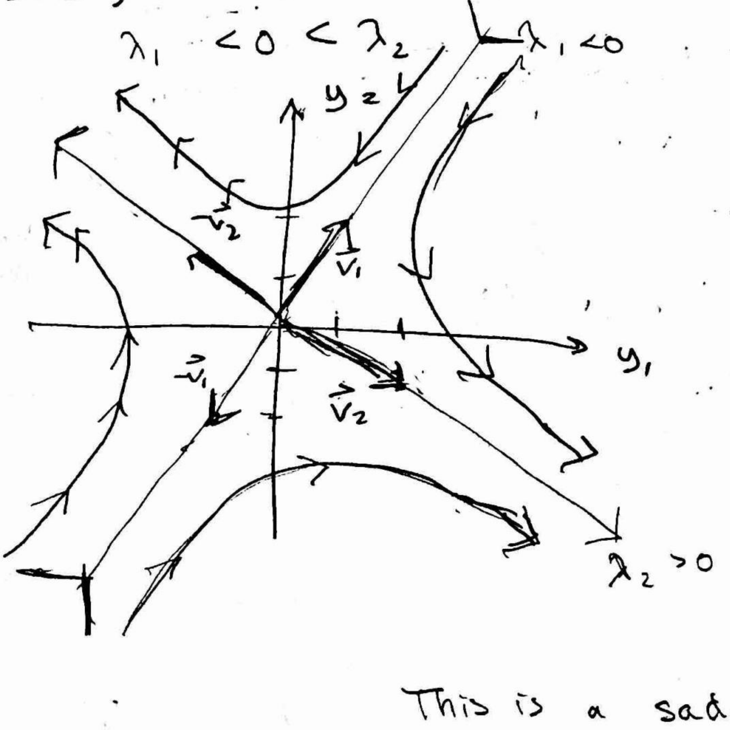
$(A - 2I) = \begin{pmatrix} -4 & -2 \\ 2 & 1 \end{pmatrix}$

$\vec{V}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\vec{y}_2(t) = e^{2t}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$$\boxed{y_{general} = C_1 e^{-t}\begin{pmatrix} 2 \\ -1 \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} -1 \\ 2 \end{pmatrix}}$$

Q3. (b)



As $t \to -\infty$, $C_2 e^{2t}\vec{v}_2 \to 0$, solution approaches $C_1 e^{-t}\vec{v}_1$.

As $t \to \infty$, $C_1 e^{-t}\vec{v}_1 \to 0$, solution approaches $C_2 e^{2t}\vec{v}_2$.

This is a saddle point, w/ 2 stable & 2 unstable halflines.

3.1 3 (a) **10 / 10**

- ✓ **- 0 pts** Correct
- **- 4 pts** Miscomputed one eigenvector.
- **- 2 pts** You found fundamental solutions, but forgot C_1 and C_2.
- **- 3 pts** Miscomputed both eigenvectors.
- **- 1 pts** Write down the general solution, not just fundamental solutions.

Name: Thilan Tran   ID: 605140530

Q3.(a)

$\vec{y}' = A\vec{y}, \quad A = \begin{pmatrix} -2 & -2 \\ 2 & 3 \end{pmatrix}$

$T = -2 + 3 = +1$

$D = -6 - (-4)$
$= -6 + 4 = -2$

$\lambda^2 - T\lambda + D = 0$

$\lambda^2 - \lambda - 2 = 0$

$(\lambda - 2)(\lambda + 1) = 0$

$\lambda_1 = -1$:

$(A + 1I) = \begin{pmatrix} -1 & -2 \\ 2 & 4 \end{pmatrix} \cdot \vec{v}_1 = \vec{0}$

$\vec{v}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\vec{y}_1(t) = e^{-t}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\lambda_2 = 2$:

$(A - 2I) = \begin{pmatrix} -4 & -2 \\ 2 & 1 \end{pmatrix}$

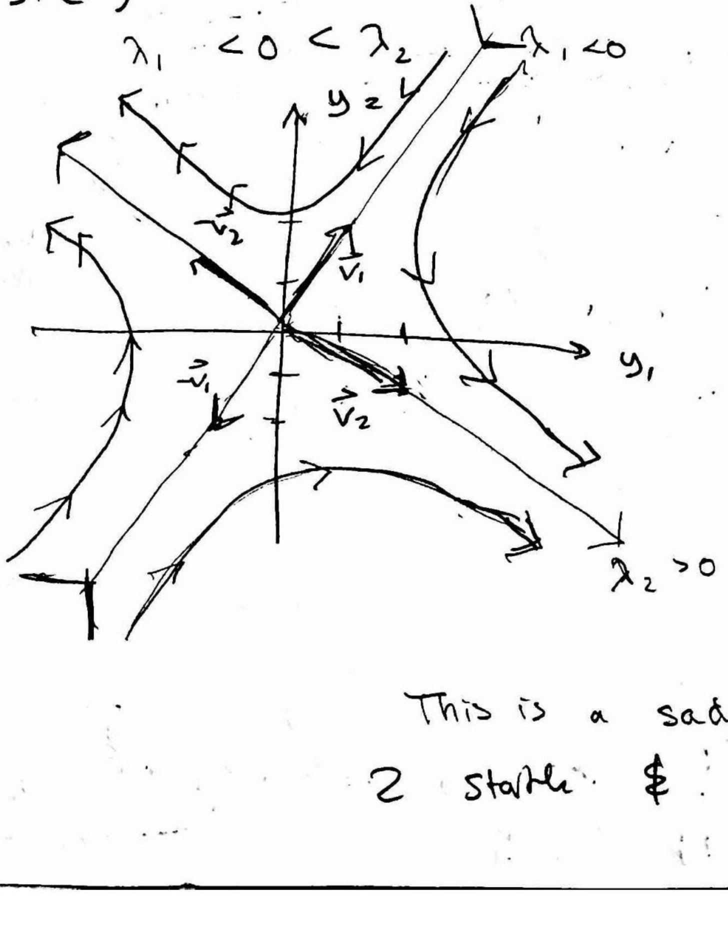
$\vec{v}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\vec{y}_2(t) = e^{2t}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$$\boxed{y_{general} = C_1 e^{-t}\begin{pmatrix} 2 \\ -1 \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} -1 \\ 2 \end{pmatrix}}$$

Q3. (b)

$\lambda_1 < 0 < \lambda_2$



As $t \to -\infty$, $C_2 e^{2t} \vec{v}_2 \to 0$, solution approaches $C_1 e^{-t} \vec{v}_1$.

As $t \to \infty$, $C_1 e^{-t} \vec{v}_1 \to 0$, solution approaches $C_2 e^{2t} \vec{v}_2$.

This is a saddle point, w/ 2 stable & 2 unstable halflines.

3.2 3(b) **3 / 5**

- **- 0 pts** Correct
- **- 1 pts** Justification?
- ✓ **- 2 pts** **Eigenvectors graphed in incorrect quadrants.**
- **- 2 pts** Indicate direction travelled on solution curves.
- **- 2 pts** The shape of your curves as t goes to infinity or - infinity is wrong
- **- 3 pts** Draw solution curves in quadrants cut out by eigenvectors.
- **- 3 pts** Draw the half-line solutions (the ones corresponding to the eigenvectors).

Name : Thilan Tran   ID : 605140530

Q4.

$$\vec{y}' = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 2 \end{pmatrix} \vec{y}, \quad \vec{y}(0) = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}.$$

$$\det\begin{pmatrix} 3-\lambda & 1 & 1 \\ -1 & 1-\lambda & -1 \\ 1 & 1 & 2-\lambda \end{pmatrix} = (3-\lambda)(2-3\lambda+\lambda^2+1) - 1(-2+\lambda+1) + 1(-1-1+\lambda)$$

$$= 9 - 9\lambda + 3\lambda^2 - 3\lambda + 3\lambda^2 - \lambda^3 + 1 - \lambda - 2 + \lambda$$

$$= -\lambda^3 + 6\lambda^2 + 12\lambda + 8$$

$$= \lambda^3 + 6\lambda^2 + 12\lambda - 8$$

Need 3 zeros.

factors ↓ ± 1 ↓ factors $\pm 1, \pm 2, \pm 4, \pm 8$

Check 1: $1 - 6 + 12 - 8 \neq 0$.

-1: $-1 - 6 - 12 - 8 \neq 0$

2: $8 - 24 + 24 - 8 = 0$ ✓

8:

⇓

factor...

$$\begin{array}{r} \lambda^2 - 4\lambda + 4 \\ (\lambda - 2)\overline{)\lambda^3 - 6\lambda^2 + 12\lambda - 8} \\ -\lambda^3 + 2\lambda^2 \\ \hline -4\lambda^2 + 12\lambda \\ +4\lambda^2 - 8\lambda \\ \hline 4\lambda - 8 \\ -4\lambda + 8 \\ \hline 0 \end{array}$$

$(\lambda^2 - 4\lambda + 4) = (\lambda - 2)(\lambda - 2)$

$\lambda = 2$ is only root w/ multiplicity 3.

$$A - 2I = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \Rightarrow \text{nullspace has dim 1}$$

$$(A-2I)^2 = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

⇒ nullspace has dim 2

$$(A-2I)^3 = \begin{pmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \text{nullspace has dim 3}$$

$(A - 2I)\cdot\vec{v}_1 = \vec{0}$

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\vec{y}_1(t) = e^{tA}\vec{v}_1 = e^{2t}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$(A - 2I)^2 \cdot \vec{v}_2 = \vec{0}$

$$\vec{v}_2 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

(linearly independent from $\vec{v}_1$) ✓

$$\vec{y}_2(t) = e^{tA}\vec{v}_2 = e^{2t}(\vec{v}_2 + t(A-2I)\vec{v}_2)$$

$$= e^{2t}\left(\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}\right)$$

Maps all of $\mathbb{R}^3$ to $\vec{0}$, so we can choose $\vec{v}_3$ to be any vector lin. independent from $\vec{v}_1$ & $\vec{v}_2$.

Choose $\vec{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (linearly independent) ✓

CONTINUED →

Name: Thilan Tran ID: 605140530

Q4 (con.):

$$\vec{y_3}(t) = e^{tA}\vec{v_3} = e^{2t}\left(\vec{v_3} + t[A-2I]\vec{v_3} + \frac{t^2}{2}[A-2I]^2\vec{v_3}\right)$$

$$= e^{2t}\left(\begin{pmatrix}0\\1\\0\end{pmatrix} + t\begin{pmatrix}1\\-1\\0\end{pmatrix} + \frac{t^2}{2}\begin{pmatrix}1\\-1\\0\end{pmatrix}\right)$$

$$\vec{y}(t) = c_1\vec{y_1}(t) + c_2\vec{y_2}(t) + c_3\vec{y_3}(t)$$

$$\vec{y}(0) = c_1\begin{pmatrix}-1\\-1\\0\end{pmatrix} + c_2\begin{pmatrix}1\\-1\\2\end{pmatrix} + c_3\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}2\\-2\\1\end{pmatrix}$$

$-c_1 + c_2 = 2$

$-c_1 - c_2 + c_3 = -2$

$2c_2 = 1$

$c_2 = \frac{1}{2}$, $c_1 = -\frac{3}{2}$

$\frac{3}{2} - \frac{1}{2} + c_3 = -2$

$c_3 = 0$

$$\boxed{\vec{y}(t) = +\frac{3}{2}e^{2t}\begin{pmatrix}-1\\-1\\0\end{pmatrix} + \frac{1}{2}e^{2t}\begin{pmatrix}1+2t\\-1-2t\\2\end{pmatrix}}$$

Q5:

$$\vec{y}' = \begin{pmatrix}0&0&0&0&0&0\\0&2&1&0&0&0\\0&-1&0&0&0&0\\0&0&0&1&0&0\\0&0&0&0&0&-2\\0&0&0&0&2&0\end{pmatrix}\vec{y}$$

Use block matrices:

$A = \begin{pmatrix}0&0&0\\0&2&1\\0&-1&0\end{pmatrix}$, $B = (1)$, $C = \begin{pmatrix}0&-2\\2&0\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}A&0&0\\0&B&0\\0&0&C\end{pmatrix}$$

A Solution:

$$(A - \lambda I) = \begin{pmatrix}-\lambda&0&0\\0&2-\lambda&1\\0&-1&-\lambda\end{pmatrix}$$

$\det(A - \lambda I) = -\lambda(-2\lambda + \lambda^2 + 1)$

$= -\lambda^3 + 2\lambda^2 - \lambda = 0$

$= \lambda(-\lambda^2 + 2\lambda + 1) = 0$

$\lambda(\lambda - 1)^2 = 0$

$\lambda_1 = 0$:

$(A - 0I) = \begin{pmatrix}0&0&0\\0&2&1\\0&-1&0\end{pmatrix}$, $\vec{v_1} = \vec{0}$

$\vec{v_1} = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $\vec{a_1} = e^{0t}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$

$\lambda_2 = 1$:

$(A - I) = \begin{pmatrix}-1&0&0\\0&1&1\\0&-1&-1\end{pmatrix}$, $\vec{v_2} = \vec{0}$

$\vec{v_2} = \begin{pmatrix}0\\1\\-1\end{pmatrix}$

$\vec{a_2} = e^t\begin{pmatrix}0\\1\\-1\end{pmatrix}$

$(A-I)^2 = \begin{pmatrix}-1&0&0\\0&1&1\\0&-1&-1\end{pmatrix}\begin{pmatrix}-1&0&0\\0&1&1\\0&-1&-1\end{pmatrix} = \begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}$ nullspace has dim = 2!

Pick $\vec{v_3}$ linearly independent from $\vec{v_1}$ & $\vec{v_2}$.

$\vec{v_3} = \begin{pmatrix}0\\0\\1\end{pmatrix}$ $\vec{a_3} = e^t(\vec{v_3} + t(A-I)\vec{v_3})$

CONTINUED →

4 Question 4 **10 / 10**

- ✓ **- 0 pts** **Correct**
- **- 2 pts** Your third fundamental solution is wrong/missing.
- **- 2 pts** Your second fundamental solution is wrong/missing.
- **- 1 pts** e^{2t} not e^t.
- **- 2 pts** System of coefficients solved incorrectly/solution to IVP missing/incorrect.
- **- 2 pts** Your first fundamental solution is wrong/missing.

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Q4 (con.):

$$\vec{y_3}(t) = e^{tA}\vec{v_3} = e^{2t}\left(\vec{v_3} + t[A-2I]\vec{v_3} + \frac{t^2}{2}[A-2I]^2\vec{v_3}\right)$$

$$= e^{2t}\left(\begin{pmatrix}0\\1\\0\end{pmatrix} + t\begin{pmatrix}1\\-1\\1\end{pmatrix} + \frac{t^2}{2}\begin{pmatrix}1\\-1\\0\end{pmatrix}\right)$$

$$\vec{y}(t) = c_1\vec{y_1}(t) + c_2\vec{y_2}(t) + c_3\vec{y_3}(t)$$

$$\vec{y}(0) = c_1\begin{pmatrix}-1\\-1\\0\end{pmatrix} + c_2\begin{pmatrix}1\\-1\\2\end{pmatrix} + c_3\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}2\\-2\\1\end{pmatrix}$$

$-c_1 + c_2 = 2$

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$c_2 = \frac{1}{2}$, $c_1 = -\frac{3}{2}$

$\frac{3}{2} - \frac{1}{2} + c_3 = -2$

$c_3 = 0$

$$\boxed{\vec{y}(t) = +\frac{3}{2}e^{2t}\begin{pmatrix}-1\\-1\\0\end{pmatrix} + \frac{1}{2}e^{2t}\begin{pmatrix}1+2t\\-1-2t\\2\end{pmatrix}}$$

Q5: $$\vec{y}\,' = \begin{pmatrix}0&0&0&0&0&0\\0&2&1&0&0&0\\0&-1&0&0&0&0\\0&0&0&1&0&0\\0&0&0&0&0&-2\\0&0&0&0&2&0\end{pmatrix}\vec{y}$$

Use block matrices:

$A = \begin{pmatrix}0&0&0\\0&2&1\\0&-1&0\end{pmatrix}$, $B = (1)$, $C = \begin{pmatrix}0&-2\\2&0\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}A&0&0\\0&B&0\\0&0&C\end{pmatrix}$$

A Solution:

$$(A-\lambda I) = \begin{pmatrix}-\lambda&0&0\\0&2-\lambda&1\\0&-1&-\lambda\end{pmatrix}$$

$$\det(A-\lambda I) = -\lambda(-2\lambda+\lambda^2+1)$$

$$= -\lambda^3 + 2\lambda^2 - \lambda = 0$$

$$= \lambda(-\lambda^2 + 2\lambda + 1) = 0$$

$$\lambda(\lambda-1)^2 = 0$$

$\lambda_1 = 0$:

$(A - 0I) = \begin{pmatrix}0&0&0\\0&2&1\\0&-1&0\end{pmatrix} \cdot \vec{v_1} = \vec{0}$

$\vec{v_1} = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $\vec{a_1} = e^{0t}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$

$\lambda_2 = 1$:

$(A - I) = \begin{pmatrix}-1&0&0\\0&1&1\\0&-1&-1\end{pmatrix} \cdot \vec{v_2} = \vec{0}$

$\vec{v_2} = \begin{pmatrix}0\\1\\-1\end{pmatrix}$

$\vec{a_2} = e^t\begin{pmatrix}0\\1\\-1\end{pmatrix}$

$(A-I)^2 = \begin{pmatrix}-1&0&0\\0&1&1\\0&-1&-1\end{pmatrix}\begin{pmatrix}-1&0&0\\0&1&1\\0&-1&-1\end{pmatrix} = \begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}$ nullspace has dim = 2!

Pick $\vec{v_3}$ linearly independent from $\vec{v_1}$ & $\vec{v_2}$.

$\vec{v_3} = \begin{pmatrix}0\\0\\1\end{pmatrix}$ $\vec{a_3} = e^t(\vec{v_3} + t(A-I)\vec{v_3})$

CONTINUED →

Name: Thilan Tran   ID: 605140530

Q5 (con.).

$$\vec{a}_3 = e^t\left(\begin{pmatrix}0\\0\\1\end{pmatrix} + t\begin{pmatrix}0\\1\\-1\end{pmatrix}\right)$$

C Solution:

$$C = \begin{pmatrix}0 & -2\\ 2 & 0\end{pmatrix}$$

$$\lambda^2 - T\lambda + D = 0$$

$$\lambda^2 + 4 = 0$$

$$\lambda = \frac{0 \pm \sqrt{-16}}{2}$$

$$= \pm 2i$$

$$(A - 2iI) = \begin{pmatrix}-2i & -2\\ 2 & -2i\end{pmatrix} \cdot \vec{w} = 0$$

$$\vec{w} = \begin{pmatrix}-1\\ i\end{pmatrix}$$

$$\vec{c}_1 = e^{2it}\begin{pmatrix}-1\\ i\end{pmatrix}$$

$$\vec{c}_2 = \overline{\vec{c}_1} = e^{-2it}\begin{pmatrix}-1\\ -i\end{pmatrix}$$

The fundamental set is:

$$\left\{ \begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix}, e^t\begin{pmatrix}0\\1\\-1\\0\\0\\0\end{pmatrix}, e^t\begin{pmatrix}0\\t\\1-t\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\e^t\\0\\0\end{pmatrix}, e^{2it}\begin{pmatrix}0\\0\\0\\0\\-1\\i\end{pmatrix}, e^{-2it}\begin{pmatrix}0\\0\\0\\0\\-1\\-i\end{pmatrix} \right\}$$

B Solution:

$$B = (1)$$

$$\cancel{(B - \lambda I) \quad 1 - \lambda = 0}$$

$$\cancel{\lambda = -1}$$

$$\cancel{(B + I)\vec{v} = (2)}$$

$$\vec{y}' = (1)y$$

$$\vec{y}' = y$$

$$\int \frac{y'}{y} = \int 1$$

$$e^{\ln(y)} = e^t$$

$$y = e^t$$

$$\vec{b} = e^t$$

Can fill out fundamental set depending on location of block matrices:

$$\begin{pmatrix}A & 0 & 0\\ 0 & B & 0\\ 0 & 0 & C\end{pmatrix}$$

$$\vec{a}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \Rightarrow \begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix}$$

$$\vec{a}_2(t) = e^t\begin{pmatrix}0\\1\\-1\end{pmatrix} \Rightarrow e^t\begin{pmatrix}0\\1\\-1\\0\\0\\0\end{pmatrix}$$

$$\vec{a}_3(t) = e^t\begin{pmatrix}0\\t\\1-t\end{pmatrix} \Rightarrow e^t\begin{pmatrix}0\\t\\1-t\\0\\0\\0\end{pmatrix}$$

$$\vec{b} = e^t \Rightarrow \begin{pmatrix}0\\0\\0\\e^t\\0\\0\end{pmatrix}$$

$$\vec{c}_1 = e^{2it}\begin{pmatrix}-1\\ i\end{pmatrix} \Rightarrow e^{2it}\begin{pmatrix}0\\0\\0\\0\\-1\\i\end{pmatrix}$$

$$\vec{c}_2 = e^{-2it}\begin{pmatrix}-1\\ -i\end{pmatrix} \Rightarrow e^{-2it}\begin{pmatrix}0\\0\\0\\0\\-1\\-i\end{pmatrix}$$

5 Question 5 **13 / 15**

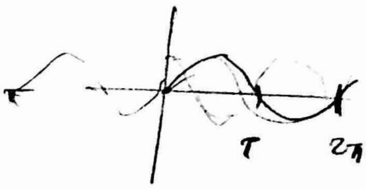
- **- 0 pts** Correct
- **- 2 pts** Identify block matrices
- **- 3 pts** Find eigenvalues for each block
- **- 3 pts** Find (generalized) eigenvectors for each block
- **- 4 pts** Construct solutions for each block
- **- 3 pts** Combine solutions.
- **- 1 pts** Minor calculation error
- ✓ **- 2 pts Moderate error in solution for one block**

We want real valued solutions

Name: Thilan Tran ID: 605140530

Q6.: (a)

$$y' = \underbrace{\sin y + \cos y}_{f(y)}$$



Set $f(y) = 0$

$\sin y + \cos y = 0.$

$\Downarrow$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \sin\left(\frac{\pi}{4}\right)$

$\sin(y + a) = \sin y \cos a + \cos y \sin a$

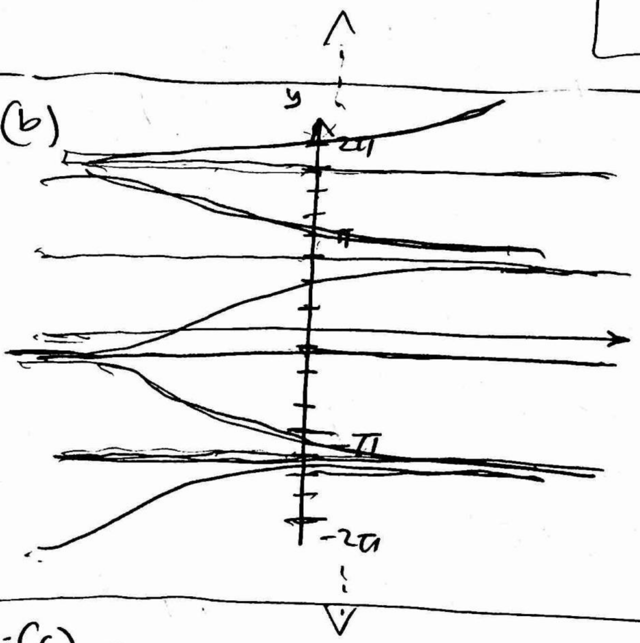
$= \frac{2}{\sqrt{2}}\left(\sin y \cos\left(\frac{\pi}{4}\right) + \cos y \sin\left(\frac{\pi}{4}\right)\right)$

$f(y) = \frac{2}{\sqrt{2}} \sin\left(y + \frac{\pi}{4}\right) = 0$

zeroes at $y = k\pi - \frac{\pi}{4}$

$$\boxed{\text{eq. points} = \ldots -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4} \text{ etc.} = k\pi - \frac{\pi}{4} \quad \forall k \in \mathbb{Z}}$$

Q6: (b)



$f(2\pi) = \sin\left(2\pi + \frac{\pi}{4}\right) > 0$

$f(\pi) = \sin\left(\pi + \frac{\pi}{4}\right) < 0$

$f(0) = \sin\left(0 + \frac{\pi}{4}\right) > 0$

$f(-\pi) = \sin\left(-\pi + \frac{\pi}{4}\right) < 0$

$f(-2\pi) = \sin\left(-2\pi - \frac{\pi}{4}\right) > 0$

Q6: (c) Prove if $y(t)$ is a solution, then $y(t)$ is a bounded solution.

Since $f(y) = \sin y + \cos y$ and $f'(y) = \cos y - \sin y$ are defined and continuous $\forall \mathbb{R}$, the uniqueness theorem is true. Thus any unique solution curves cannot cross by the theorem. Thus, since the equilibrium solutions for y' occur periodically from $y \in (-\infty, \infty)$, if $y(t)$ were not bounded it would cross one of these equilibrium solutions. The bounds in question are the closest pair of $k\pi - \pi/4$ $\forall k \in \mathbb{Z}$.

6.1 6(a) **5 / 5**

✓ **- 0 pts** **Correct**

- 2 pts Knew to find zeros of RHS.

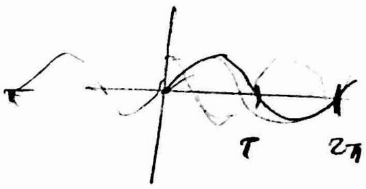
- 2 pts Correctly found at least one infinite family of solutions.

- 1 pts Found half of the solutions or made a computational mistake.

Name: Thilan Tran   ID: 605140530

Q6.: (a)

$$y' = \underbrace{\sin y + \cos y}_{f(y)}$$



Set $f(y) = 0$

$\sin y + \cos y = 0.$

$\Downarrow$

$\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} = \sin(\frac{\pi}{4})$

$$\sin(y + a) = \sin y \cos a + \cos y \sin a$$

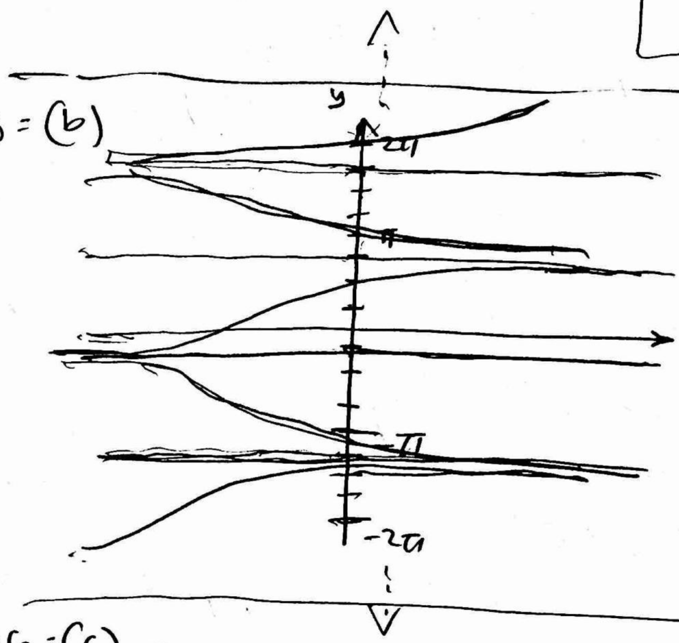
$$= \frac{2}{\sqrt{2}} \left(\sin y \cos(\tfrac{\pi}{4}) + \cos y \sin(\tfrac{\pi}{4}) \right)$$

$$f(y) = \frac{2}{\sqrt{2}} \sin(y + \tfrac{\pi}{4}) = 0$$

zeroes at $y = k\pi - \frac{\pi}{4}$

$$\boxed{\text{eq. points} = \ldots -\tfrac{\pi}{4}, \tfrac{3\pi}{4}, \tfrac{7\pi}{4} \ldots \text{etc.} = k\pi - \tfrac{\pi}{4} \ \forall k \in \mathbb{Z}}$$

Q6: (b)



$f(2\pi) = \sin(2\pi + \frac{\pi}{4}) > 0$

$f(\pi) = \sin(\pi + \frac{\pi}{4}) < 0$

$f(0) = \sin(0 + \frac{\pi}{4}) > 0$

$f(-\pi) = \sin(-\pi + \frac{\pi}{4}) < 0$

$f(-2\pi) = \sin(-2\pi - \frac{\pi}{4}) > 0$

Q6: (c) Prove if $y(t)$ is a solution, then $y(t)$ is a bounded solution.

Since $f(y) = \sin y + \cos y$ and $f'(y) = \cos y - \sin y$ are defined and continuous $\forall \mathbb{R}$, the uniqueness theorem is true. Thus any unique solution curves cannot cross by the theorem. Thus, since the equilibrium solutions for y' occur periodically from $y \in (-\infty, \infty)$, if $y(t)$ were not bounded it would cross one of these equilibrium solutions. The bounds in question are the closest pair of $k\pi - \pi/4 \ \forall k \in \mathbb{Z}$.

6.2 6(b) **5 / 5**

✓ **- 0 pts** Correct

- 2 pts Included equilibria

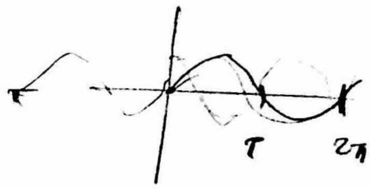
- 3 pts Solutions go in correct directions

- 1 pts Violates uniqueness

- 1 pts Small error

Name: Thilan Tran ID: 605140530

Q6. (a) $y' = \underbrace{\sin y + \cos y}_{f(y)}$



Set $f(y) = 0$

$\sin y + \cos y = 0.$

$\Downarrow$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \sin\left(\frac{\pi}{4}\right)$

$\sin(y + a) = \sin y \cos a + \cos y \sin a$

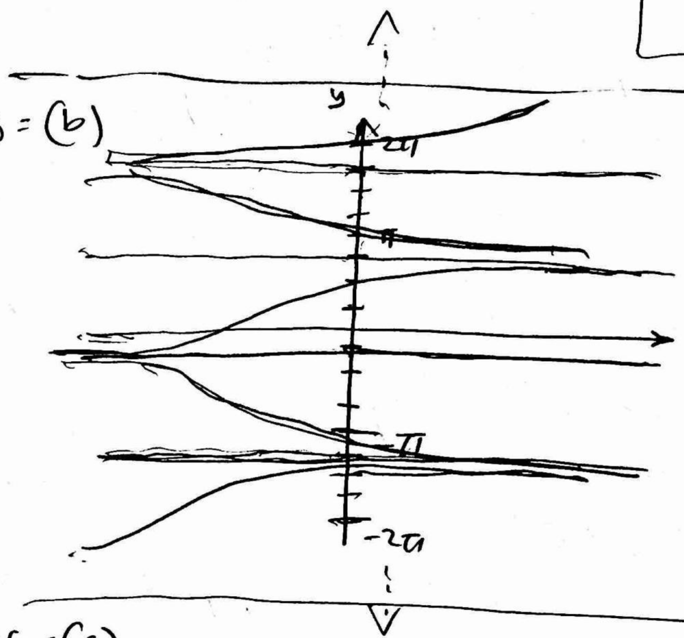
$= \frac{2}{\sqrt{2}}\left(\sin y \cos\left(\frac{\pi}{4}\right) + \cos y \sin\left(\frac{\pi}{4}\right)\right)$

$f(y) = \frac{2}{\sqrt{2}} \sin\left(y + \frac{\pi}{4}\right) = 0$

zeroes at $y = k\pi - \frac{\pi}{4}$

$\boxed{\text{eq. points } = \ldots -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4} \text{ etc.} = k\pi - \frac{\pi}{4} \ \forall k \in \mathbb{Z}}$

Q6: (b)



$f(2\pi) = \sin\left(2\pi + \frac{\pi}{4}\right) > 0$

$f(\pi) = \sin\left(\pi + \frac{\pi}{4}\right) < 0$

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Since $f(y) = \sin y + \cos y$ and $f'(y) = \cos y - \sin y$ are defined and continuous $\forall \mathbb{R}$, the uniqueness theorem is true. Thus any unique solution curves cannot cross by the theorem. Thus, since the equilibrium solutions for y' occur periodically from $y \in (-\infty, \infty)$, if $y(t)$ were not bounded it would cross one of these equilibrium solutions. The bounds in question are the closest pair of $k\pi - \pi/4 \ \forall k \in \mathbb{Z}$.

6.3 6(c) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 2 pts** Invoke hypotheses of existence and uniqueness.
- **- 3 pts** Bound by equilibria
- **- 1 pts** Minor error
- **- 1 pts** Show it satisfies the hypotheses (show f and f' are continuous).
- **- 1 pts** State that there are arbitrarily large or small equilibria.
- **- 1 pts** Invoke existence and uniqueness.

Name: Thilan Tran ID: 605140330

Q7(a.): $a > 0$. First order, linear, nonhomogenous...

$$(t^2 - a^2)\, y' = y + t^2 - t - a^2$$

$$y' - \left(\frac{1}{t^2-a^2}\right) y = \overbrace{\frac{t^2 - t - a^2}{t^2 - a^2}}^{h}$$

$$y' = f(t,y) = \frac{1}{t^2-a^2}\, y + t^2 - t - a^2$$

$$u = e^{-\int a(t)\,dt}$$

$$\frac{1}{t^2-a^2} = \frac{A}{t-a} + \frac{B}{t+a}$$

$$1 = At + Aa + Bt - Ba$$

$$A + B = 0$$
$$A - B = \frac{1}{a}$$
$$2A = \frac{1}{a}$$
$$A = \frac{1}{2a}$$
$$B = \frac{-1}{2a}$$

$$\frac{1}{a^2 - t^2} = \frac{A}{a-t} + \frac{B}{a+t}$$

$$1 = Aa + At + Ba - Bt$$
$$A - B = 0$$
$$Aa + Ba = 1$$

$$= e^{-\int \frac{1}{t^2-a^2}\,dt}$$

$$= e^{-\int \frac{1}{2a(t-a)} - \frac{1}{2a(t+a)}\,dt}$$

$$= e^{+\left[\frac{1}{2a}\ln|t-a| + \frac{1}{2a}\ln|t+a|\right]}$$

$$= e^{\frac{1}{2a}\ln\frac{|t+a|}{|t-a|}} = \left(\frac{t+a}{t+a}\right)^{-\frac{1}{2}a}$$

$$= \sqrt{\left(\frac{t+a}{t-a}\right)^a}$$

$t^2 - t - a^2$

$(t-a)^2 + 2at + t$

$$\Rightarrow uy = \int u f\,dt + c$$

$$= \int \left(t^2 - t - \frac{t}{t^2-a^2}\right) \cdot \frac{(t+a)^{\frac{1}{2}a}}{(t-a)^{\frac{1}{2}a}}$$

$$\frac{t-a}{t+a} = 1 + \frac{2a}{t+a}$$

?

Q7(b.)

Q7(c.). Note that $f(t,y)$ is continuous except when $t^2 - a^2 = 0$, or $t = \pm a$. Thus, at a the existence and uniqueness theorem does not hold, which allows for the strange possibility of infinitely many solutions to the IVP here.

7.1 7(a) 1 / 5

- **- 0 pts** Correct
- **- 2 pts** y_h /IF correct, didn't find y_p
- **- 2 pts** minor mistake / gap
- ✓ **- 4 pts** **Major mistake/gap**
- **- 5 pts** blank
- **- 3 pts** y_h/IF minor mistake/not simplified, didn't find y_p
- **- 1 pts** y_h not simplifed
- **- 3 pts** y_h = ?
- **- 1 pts** typo

Name: Thilan Tran ID: 605140330

Q7(a.): $a > 0$. First order, linear, inhomogenous...

$$(t^2 - a^2) y' = y + t^2 - t - a^2$$

$$y' - \left(\frac{1}{t^2-a^2}\right) y = \overbrace{\frac{t^2 - t - a^2}{t^2-a^2}}^{h}$$

$y' = f(t,y)$
$= \frac{1}{t^2-a^2} y + t^2 - t - a^2$

$$u = e^{-\int a(t)\,dt}$$

$\frac{1}{t^2-a^2} = \frac{A}{t-a} + \frac{B}{t+a}$

$1 = At + Aa + Bt - Ba$

$A + B = 0$

$A - B = \frac{1}{a}$

$2A = \frac{1}{a}$

$A = \frac{1}{2a}$

$B = -\frac{1}{2a}$

$\frac{1}{a^2-t^2} = \frac{A}{a-t} + \frac{B}{a+t}$

$1 = Aa + At + Ba - Bt$

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$$= e^{-\int \frac{1}{t^2-a^2}\,dt}$$

$$= e^{-\int \frac{1}{2a(t-a)} - \frac{1}{2a(t+a)}\,dt}$$

$$= e^{+\left[\frac{1}{2a}\ln|t-a| + \frac{1}{2a}\ln|t+a|\right]}$$

$t^2 - t - a^2$

$(t-a)^2 + 2at + t$

$$= e^{\frac{1}{2a}\ln\frac{|t+a|}{|t-a|}} = \left(\frac{t+a}{t+a}\right)^{-\frac{1}{2}a}$$

$$\Rightarrow uy = \int u f\,dt + c$$

$$= \sqrt{\left(\frac{t+a}{t-a}\right)^a}$$

$\frac{t-a}{t+a} = 1 + \frac{2a}{t+a}$

$$= \int \left(\frac{t^2 - t}{t^2-a^2}\right) \cdot \frac{(t+a)^{\frac{1}{2}a}}{(t-a)^{\frac{1}{2}a}}$$

?

Q7(b.).

Q7(c.). Note that $f(t,y)$ is continuous except when $t^2 - a^2 = 0$, or $t = \pm a$. Thus, at a the existence and uniqness theorem does not hold, which allows for the strange possibility of infinitely many solutions to the IVP there.

7.2 7(b) **0 / 3**

- **- 0 pts** Correct:
- **- 1 pts** minor mistake (e.g. forget to say C can be anything), gap, logic flow not clear
- **- 2 pts** some meaningful writings. not much detail provided, many gaps.
- ✓ **- 3 pts** nothing meaningful

Name: Thilan Tran ID: 605140330

Q7(a.): $a > 0$. First order, linear, inhomogenous...

$$(t^2 - a^2)y' = y + t^2 - t - a^2$$

$$y' - \left(\frac{1}{t^2-a^2}\right)y = \underbrace{\frac{t^2-t-a^2}{t^2-a^2}}_{h}$$

$y' = f(t,y)$
$= \frac{1}{t^2-a^2}y + t^2 - t - a^2$

$$u = e^{-\int a(t)\,dt}$$

$\frac{1}{t^2-a^2} = \frac{A}{t-a} + \frac{B}{t+a}$

$1 = At + Aa + Bt - Ba$

$A + B = 0$

$A - B = \frac{1}{a}$

$2A = \frac{1}{a}$

$A = \frac{1}{2a}$

$B = -\frac{1}{2a}$

$\frac{1}{a^2-t^2} = \frac{A}{a-t} + \frac{B}{a+t}$

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$A - B = 0$

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$$= e^{-\int \frac{1}{t^2-a^2}\,dt}$$

$$= e^{-\int \frac{1}{2a(t-a)} - \frac{1}{2a(t+a)}\,dt}$$

$$= e^{+\left[\frac{1}{2a}\ln|t-a| + \frac{1}{2a}\ln|t+a|\right]}$$

$t^2 - t - a^2$

$(t-a)^2 + 2at + t$

$$= e^{\frac{1}{2a}\ln\frac{|t+a|}{|t-a|}} = \left(\frac{t+a}{t-a}\right)^{-\frac{1}{2}a}$$

$$= \sqrt{\left(\frac{t+a}{t-a}\right)^a}$$

$\frac{t-a}{t+a} = 1 + \frac{2a}{t+a}$

$$\Rightarrow uy = \int u\,h\,dt + c$$

$$= \int \left(\frac{t^2-t}{t^2-a^2}\right) \cdot \frac{(t+a)^{\frac{1}{2}a}}{(t-a)^{\frac{1}{2}a}}$$

?

Q7(b.)

Q7(c.). Note that $f(t,y)$ is continuous except when $t^2 - a^2 = 0$, or $t = \pm a$. Thus, at a the existence and uniqueness theorem does not hold, which allows for the strange possibility of infinitely many solutions to the IVP there.

7.3 7(c) **0 / 2**

- **- 0 pts** Correct
- **- 2 pts** wrong
- ✓ **- 1 pts didn't put in normal form y' = F(y,t) = 1/ (t^2 - a^2) y + ...**
- ✓ **- 1 pts didn't check/state that \partial F/ \partial y = 1/(t^2 - a^2) or calculation is wrong**
- **- 1 pts** Gap

Name: Thilan Tran   ID: 605140530

Q8:

Find e^{tA} where $A = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$.

Rewrite $A = aI + b\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. (from homework hint in textbook)

$$e^{tA} = e^{taI} \cdot e^{tb\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}$$

$$e^{taI} = e^{at} \cdot I$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

By truncation, $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}^k$ for $k > 2$ is also $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

$$e^{tb\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}} = I + tb\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \cancel{\dots} \; 0$$

$$= \begin{pmatrix} 1 & tb \\ 0 & 1 \end{pmatrix}$$

$$e^{tA} = e^{at} \cdot I \cdot \begin{pmatrix} 1 & tb \\ 0 & 1 \end{pmatrix}$$

$$\boxed{e^{tA} = e^{at} \cdot \begin{pmatrix} 1 & tb \\ 0 & 1 \end{pmatrix}}$$

8 Question 8 **10 / 10**

- ✓ **- 0 pts** Correct
- **- 4 pts** gap: did not verify (A-aI)^2 = 0
- **- 4 pts** minor mistake
- **- 2 pts** lack essential detail / some typos
- **- 4 pts** based on your flow, you didn't use math induction to give a proof for A^n
- **- 8 pts** Major mistake