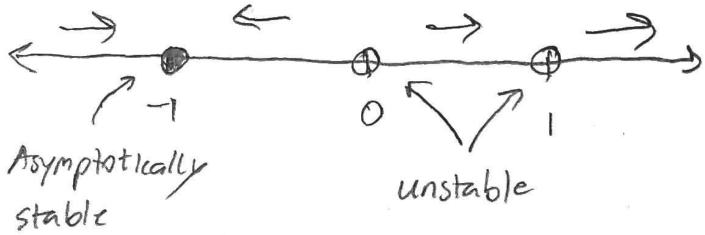
James Youn

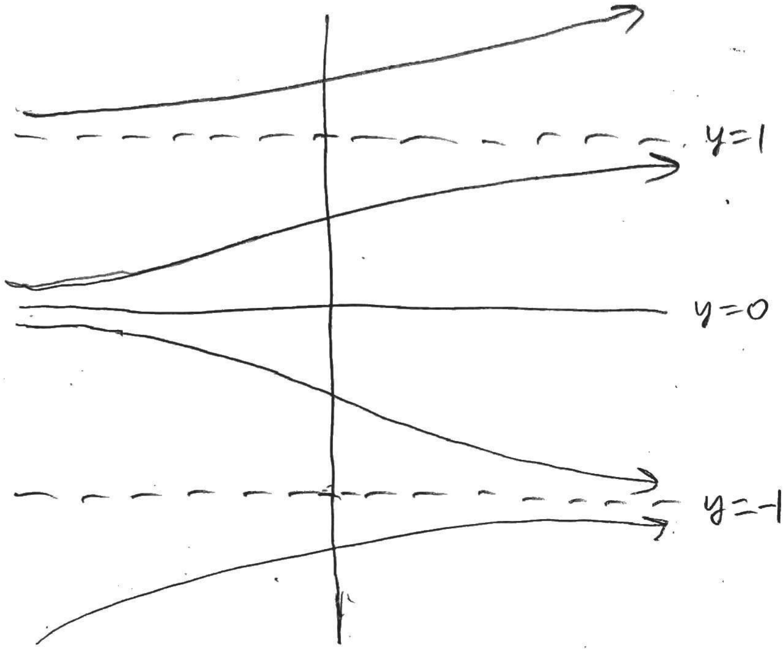
505399332

Math 33B midterm 2

1)1)



1)2)



1)3) case $-\infty < y_0 < -1$: $y \to -1$

$-1 < y_0 < 0$: $y \to -1$

$0 < y_0 < 1$: $y \to 1$

$1 < y_0 < \infty$: $y \to \infty$

2) 1)

$p^2+4p+4=0$, $(p+2)^2=0$ $\boxed{y=Ae^{-2t}+Bte^{-2t}}$

2) 2)

$y_1=e^{-2t}$, $y_2=te^{-2t}$

$\det\begin{bmatrix} y_1 & y_2 \\ y_1' & y_2' \end{bmatrix} = e^{-2t}\left[-2te^{-2t}+e^{-2t}\right] - te^{-2t}\left(-2e^{-2t}\right) =$

$-2te^{-4t}+e^{-4t}+2te^{-4t} = e^{-4t} = W(t) \neq 0$

$v_1 = \int \frac{-te^{-2t}\left(t^{-2}e^{-2t}\right)}{e^{-4t}}\,dt = \int -\frac{1}{t}\,dt = -\ln|t|$

$v_2 = \int \frac{e^{-2t}\left(t^{-2}e^{-2t}\right)}{e^{-4t}}\,dt = \int t^{-2}\,dt = -\frac{1}{t}$

$y_p = -\ln|t|e^{-2t} - e^{-2t}$

$y_g = y + y_p$ $\therefore$ $Ae^{-2}+Be^{-2}-e^{-2} = (A+B-1)e^{-2} = 0$, $\underline{A+B=1}$

$y' = -2Ae^{-2t} + B\left[-2te^{-2t}+e^{-2t}\right] = -2Ae^{-2t} - 2Bte^{-2t} + Be^{-2t}$

$y_p' = -\left(\frac{e^{-2t}}{t} - 2e^{-2t}\ln|t|\right) + 2e^{-2t} = -\frac{e^{-2t}}{t} + 2e^{-2t}\ln|t| + 2e^{-2t}$

$-2Ae^{-2} - 2Be^{-2} + Be^{-2} - e^{-2} + 2e^{-2} = (-2A-B+1)e^{-2} = 0$

$\underline{-2A-B=-1}$ $\therefore$ $A=0$, $B=1$

$\boxed{y_g = te^{-2t} - \ln|t|e^{-2t} - e^{-2t}}$

3) 1)

$y := Ae^{-t}$. $y' = -Ae^{-t}$. $y'' = Ae^{-t}$

$\underline{Ae^{-t}} + 2Ae^{-t} - 3Ae^{-t} = 0$

$y := Ate^{-t}$. $y' = A[-te^{-t} + e^{-t}] = -Ate^{-t} + Ae^{-t}$

$y'' = Ate^{-t} - Ae^{-t} - Ae^{-t} = Ate^{-t} - 2Ae^{-t}$

$Ate^{-t} - 2Ae^{-t} + 2Ate^{-t} - 2Ae^{-t} - 3Ate^{-t} = -4Ae^{-t} = e^{-t}$

$A = -\frac{1}{4}$ $\therefore$ $\boxed{y_p = -\frac{1}{4}te^{-t}}$

3) 2)

$y := -\frac{1}{3}t + a$, $y' = -\frac{1}{3}$. $y'' = 0$

$+\frac{2}{3} + t - 3a = t$ $\therefore$ $a = \frac{2}{9}$ $\therefore$ $\boxed{y_p = -\frac{1}{3}t + \frac{2}{9}}$

3) 3)

By superposition in lecture notes,

$y_p = -\frac{1}{2}te^{-t} + t - \frac{2}{3}$

$p^2 - 2p - 3 = 0$, $(p-3)(p+1)$ $p = -1, 3$. $y = Ae^{-t} + Be^{3t}$

$\boxed{y_g = Ae^{-t} + Be^{3t} - \frac{1}{2}te^{-t} + t - \frac{2}{3}}$

4) 1)

$y_1' = 3t^2$. $y_1'' = 6t$

$6t^3 - 15t^3 + kt^3 = (k-9)t^3 = 0$, $\boxed{k=9}$

4) 2)

$w := y_1$. $t^2(vw)'' - 5t(vw)' + k(vw)$.

$(vw)' = v'w + w'v$. $(vw)'' = v''w + w'v' + w''v + v'w' =$

$v''w + 2v'w' + vw''$

$t^2v''w + 2t^2v'w' + t^2vw'' - 5tv'w - 5tw'v + 9vw =$

$t^5v'' + 6t^4v' + 6t^3v - 5t^4v' - 15t^3v + 9t^3v =$

$t^5v'' + t^4v' = t^3$. $w := v'$

$t^5w' + t^4w = t^3$, $w' + \frac{1}{t}w = \frac{1}{t^2}$. $u = e^{\ln|t|} = |t|$

$(tw)' = \frac{1}{t}$. $tw = \ln|t| + A$. $w = \frac{\ln|t|}{t} + \frac{A}{t}$

$v = \int \frac{\ln|t|}{t}\,dt + A\ln|t| = \frac{1}{2}(\ln|t|)^2 + A\ln|t| + B$

$\boxed{y_p = \frac{1}{2}t^3(\ln|t|)^2 + At^3\ln|t| + Bt^3}$

4) 3)

$y_g = Ay_1 + By_2 + y_p = \boxed{At^3 + By_2 + \frac{1}{2}t^3(\ln|t|)^2 + At^3\ln|t| + Bt^3}$

4) 4)

$$W = \det\begin{bmatrix} t^3 & y_2 \\ 3t^2 & y_2' \end{bmatrix} = t^3 y_2' - 3t^2 y_2 \neq 0$$

$$y_2' - \frac{3}{t} y_2 \neq 0 \,.\; u = e^{-3\ln|t|} = t^{-3}$$

$$(t^{-3} y_2)' \neq 0 \,.\; t^{-3} y_2 \neq A \,,\; y_2 \neq At^3$$

$$t^2(Ay_1'' + By_2'') - 5t(Ay_1' + By_2') + k(Ay_1 + By_2) =$$

$$6At^3 + Bt^2 y_2'' - 15At^3 - 5Bty_2' + 9At^3 + 9By_2 =$$

$$y_2 := At^2 + Bt + C \,.\; y_2' = 2At + B \,.\; y_2'' = 2A$$

$$2At^2 - 10At^2 - 5Bt + 9At^2 + 9Bt + 9C = At^2 + 4Bt + 9C = 0$$

From part 2, suppose instead that $t^5 w' + t^4 w = 0$,

$$w' + \frac{1}{t} w = 0 \,.\; tw = C \,,\; w = \frac{C}{t} \,,\; v = C\ln|t| + D$$

$$y_2 = Ct^3\ln|t| + Dt^3 \,,\; y_2' = C\left[3t^2\ln|t| + t^2\right] + 3Dt^2 =$$

$$3Ct^2\ln|t| + (C + 3D)t^2$$

$$W = \det\begin{bmatrix} y_1 & y_2 \\ y_1' & y_2' \end{bmatrix} = 3Ct^5\ln|t| + (C+3D)t^5 - \left[3Ct^5\ln|t| + 3Dt^5\right] =$$

Ct^5. If $C \neq 0$, then there exists $t_0 > 0$ such that $Ct_0^5 \neq 0$ $\therefore$

$y = Ay_1 + By_2$ is fundamental set & $\boxed{y_2 = Ct^3\ln|t| + Dt^3,\ C \neq 0}$