5. (7 pts) Consider the differential equation

$$(t+2)y' = y^{\frac{2}{3}}.$$

(a) For what points (t_0, y_0) does the Existence Theorem guarantee that a solution exists satisfying $y(t_0) = y_0$?

b

$$y' = \frac{y^{2/3}}{t+2} \quad \Rightarrow \quad f(t,y) = \frac{y^{2/3}}{t+2}$$

f is continuous on $y: [0, \infty)$, $t: \neq 2$

But since we need a rectangle around (t_0, y_0), $y=0$ is not guaranteed ?

So $0 < y < \infty$ why, $-\infty < t < \infty$ for all $t \neq 2$

(b) For what points (t_0, y_0) does the Uniqueness Theorem guarantee that there is only one solution satisfying $y(t_0) = y_0$?

$$\frac{\partial f}{\partial y} = \frac{2}{3y^{1/3}(t+2)}$$

$\frac{\partial f}{\partial y}$ is continuous for $y > 0$ and $t \neq 2$

$y \neq 0$.

$0 < y < \infty$, $-\infty < t < \infty$, $t \neq 2$ X