University of California, Los Angeles
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MATH 33B: DIFFERENTIAL EQUATIONS
MIDTERM EXAM 2

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Q1

25 Points

Check whether the following two functions $y_1(t)$ and $y_2(t)$ are linear independent by computing Wronskian.
$y_1(t) = e^t, y_2(t) = e^{-3t}$

The Wronskian of y_1 and y_2 $= y_1 y_2' - y_1' y_2$

$W(t)$

$$= e^t \cdot (-3) e^{-3t} - e^t e^{-3t}$$

$$= e^t e^{-3t} (-3-1)$$

$$= -4 e^t e^{-3t}$$

$$= -4 e^{-2t}$$

$W(t)$ is never 0, so $y_1(t)$ and $y_2(t)$ are linearly independent.

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Q2

25 Points

For the following differential equation (use characteristic polynomial), find the general solution.

$y'' + 4y' + 13y = 0$

$$\lambda_1, \lambda_2 = \frac{-4 \pm \sqrt{16-52}}{2} = \frac{-4 \pm \sqrt{-36}}{2} = \frac{-4 \pm 6i}{2} = -2 \pm 3i$$

$\lambda_1 = -2+3i$ The characteristic polynomial ~~This equation~~ has two distinct complex roots.

$\lambda_2 = -2-3i$

Let $\lambda_1 = a+bi$, $\lambda_2 = a-bi$, then $a=-2$, $b=3$.

$$y(t; c_1, c_2) = \cancel{e^{-2t}\cos(bt)c_1 + e^{-2t}\sin(bt)c_2}$$

$$\cancel{c_1 e^{-2t}} \quad c_1 e^{-2t}\cos(3t) + c_2 e^{-2t}\sin(3t)$$

~~$e^{-2t}\cos(3t) + e^{-2t}\sin(3t)$~~

~~$-2e^{-2t}\cos(3t) +$~~

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Q3

25 Points

For the following initial value problems (use characteristic polynomial), find the solution $y(t)$.

$y'' - 4y' - 5y = 0, y(1) = -1, y'(1) = -1$

$$\lambda_1, \lambda_2 = \frac{4 \pm \sqrt{16+20}}{2} = \frac{4 \pm 6}{2} = 2 \pm 3$$

$$\lambda_1 = 5$$

$$\lambda_2 = -1$$

The characteristic polynomial has two distinct real roots.

$$y(t; C_1, C_2) = C_1 e^{5t} + C_2 e^{-t}$$

$$y(1) = C_1 e^5 + C_2 e^{-1} = -1$$

$$y'(t) = 5C_1 e^{5t} + (-C_2 e^{-t})$$

$$y'(1) = 5C_1 e^5 - C_2 e^{-1} = -1$$

$$\begin{cases} C_1 e^5 + C_2 e^{-1} = -1 & ① \\ 5C_1 e^5 - C_2 e^{-1} = -1 & ② \end{cases}$$

$$y(t) = -\frac{1}{3e^5} e^{5t} - \frac{2}{3e^{-1}} e^{-t}$$

① + ② : $6C_1 e^5 = -2$

$$C_1 = -\frac{1}{3e^5}$$

① : $-\frac{1}{3} + C_2 e^{-1} = -1$

$$C_2 = -\frac{2}{3e^{-1}}$$

$$\cancel{y' = -\frac{5}{3e^5} e^{5t} + \frac{2}{3e^{-1}} e^{-t}}$$

$$\cancel{y'' = -\frac{25}{3e^5} e^{5t} - \frac{2}{3e^{-1}} e^{-t}}$$

$$\cancel{-\frac{25}{3e^5} a - \frac{2}{3e^{-1}} b + \frac{20}{3e^5} a - \frac{8}{3e^{-1}} b + \frac{5}{3e^5} a + \frac{10}{3e^{-1}} b}$$

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Q4

25 Points

Find one particular solution to the following inhomogeneous linear differential equation:

$y'' + y' - 2y = 2t.$

Hint: Use the method of undetermined coefficients to guess an appropriate trial solution $y_p(t)$.

Plug $y = 2t$ into the equation $y'' + y' - 2y = 0$:

$$0 + 2 - 4t \neq 0$$

So, $2t$ is not a solution of $y'' + y' - 2y = 0$.

$$y_p(t) = at + b$$

plug $y = y_p(t)$ into the equation $y'' + y' - 2y = 2t$:

$$\cancel{0 + (a-2)at = 2t}$$

$$0 + a - 2at - 2b = 2t$$

$$\begin{cases} a - 2b = 0 \\ -2a = 2 \end{cases} \rightarrow \begin{cases} a = -1 \\ b = -\frac{1}{2} \end{cases}$$

$$y_p(t) = -t - \frac{1}{2}$$