21F-MATH33B-2 Midterm exam 2

ERIC ZHOU

TOTAL POINTS

97 / 100

QUESTION 1

1 Q1 25 / 25

✓ **- 0 pts Correct**

- 2 pts Click here to replace this description.

- 1 pts Click here to replace this description.

- 25 pts Click here to replace this description.

- 10 pts Click here to replace this description.

- 3 pts Click here to replace this description.

- 5 pts Click here to replace this description.

QUESTION 2

2 Q2 25 / 25

✓ **+ 25 pts Correct**

+ 15 pts Write the characteristic polynomial $$\lambda^2 + 4\lambda +13$$, and find its roots $$\lambda = -2 \pm 3i$$

+ 12 pts Wrote the characteristic polynomial but found the wrong roots

+ 10 pts Use these roots to write the general (real) solution $$y(t) = C_1 e^{-2t} \cos (3t) + C_2 e^{-2t} \sin (3t)$$

+ 8 pts Instead of writing the general real solution, wrote the general complex solution $$C_1 e^{(-2 + 3i)t} + C_2 e^{(-2 - 3i)t}$$

+ 8 pts Sign error: wrote $$y(t) = C_1 e^{2t} \cos (3t) + C_2 e^{2t} \sin (3t)$$

+ 5 pts Only wrote a particular solution, such as $$y(t) =e^{-2t} \cos (3t) + e^{-2t} \sin (3t)$$

QUESTION 3

3 Q3 25 / 25

✓ **- 0 pts Correct**

- 5 pts Correct general solution; Incorrect initial value

- 8 pts Incorrect roots from characteristic polynomial

- 20 pts Incorrect method and incorrect result

- 25 pts Empty

QUESTION 4

4 Q4 22 / 25

- 0 pts Correct

✓ **- 3 pts Minor computational error**

- 5 pts Slightly less minor computational error, or a couple minor computational errors

- 14 pts Guessed the form of $$y_p(t)$$ correctly, and solved for the constants correctly, but did not understand what those constants represented

- 11 pts Guessed $$y_p(t)$$ is a polynomial, almost correct, but missing terms or of wrong degree.

- 17 pts Guessed $$y_p(t)$$ is a polynomial, but missing terms or of wrong degree. Did not know how to solve for the constants

- 17 pts Guessed the form of $$y_p(t)$$ correctly, but did not solve for the constants.

- 16 pts Guessed the form of $$y_p(t)$$ correctly, but did not know how to solve for the constants correctly.

- 6 pts When solving for the constants, did not get a constant.

- 25 pts No points (note that finding the homogenous solution is not part of the problem, as you are asked to find only one particular solution)

- 25 pts No points

1 $$+a$$, not $$-a$$

University of California, Los Angeles
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MATH 33B: DIFFERENTIAL EQUATIONS
MIDTERM EXAM 2

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Q1

25 Points

Check whether the following two functions $y_1(t)$ and $y_2(t)$ are linear independent by computing Wronskian.

$y_1(t) = e^t, y_2(t) = e^{-3t}$

$$W = \begin{vmatrix} e^t & e^{-3t} \\ e^t & -3e^{-3t} \end{vmatrix} = -3e^t e^{-3t} - e^t e^{-3t} = -4e^t e^{-3t} = -4e^{-2t}$$

$-4e^{-2t} \neq 0$ for all $t \in \mathbb{R}$, so $y_1(t)$ and $y_2(t)$ are linearly independent.

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Q2

25 Points

For the following differential equation (use characteristic polynomial), find the general solution.

$y'' + 4y' + 13y = 0$

$r^2 + 4r + 13 = 0$

$r = \frac{-4 \pm \sqrt{16 - 52}}{2} = \frac{-4 \pm 6i}{2} = -2 \pm 3i$

$y = c_1 e^{-2t}\cos(3t) + c_2 e^{-2t}\sin(3t)$

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Q3
25 Points

For the following initial value problems (use characteristic polynomial) , find the solution $y(t)$.
$y'' - 4y' - 5y = 0, y(1) = -1, y'(1) = -1$

$r^2 - 4r - 5 = 0$
$(r-5)(r+1) = 0$
$r = 5, -1$

$y = c_1 e^{5t} + c_2 e^{-t} \Rightarrow c_1 e^5 + c_2 e^{-1} = -1$

$y' = 5c_1 e^{5t} - c_2 e^{-t} \Rightarrow 5c_1 e^5 - c_2 e^{-1} = -1$

$6c_1 e^5 = -2$
$c_1 = \frac{-1}{3e^5} = -\frac{e^{-5}}{3}$

$-\frac{1}{3} + c_2 e^{-1} = -1$
$c_1 e^{-1} = -\frac{2}{3}$
$c_2 = -\frac{2}{3}e$

$y = -\frac{e^{-5}}{3}e^{5t} - \frac{2e}{3}e^{-t}$

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Q4

25 Points

Find one particular solution to the following inhomogeneous linear differential equation:

$y'' + y' - 2y = 2t.$

Hint: Use the method of undetermined coefficients to guess an appropriate trial solution $y_p(t)$.

$y_p = at + b$

$y_p' = a$

$y_p'' = 0$

1 $-2(at+b) = 2t$

$-2at - a - 2b = 2t$

$-a - 2b = 0$

$-2at = 2t$

$a = -1$

$1 - 2b = 0$

$b = 1/2$

$y_p(t) = -t + 1/2$