Q1

25 Points

For the following differential equation, show that $y_1(t)$ and $y_2(t)$ form a fundamental set of solutions for the given differential equation (check linear independent by computing Wronskian or by definition of linear independent) . Then find a solution to the initial value problems:

$y'' + 2y' - 3y = 0, y_1(t) = e^t, y_2(t) = e^{-3t}, y(0) = 1, y'(0) = -2$

Q2

25 Points

For the following initial value problems (use characteristic polynomial) , find the solution $y(t)$.

(1) $y'' + 4y' + 13y = 0, \quad y(0) = 2, \quad y'(0) = -1$

(2) $y'' - 4y' - 5y = 0, y(1) = -1, y'(1) = -1$

Q3

25 Points

Perform the following tasks for the equations below:

(1) Find a general solution of the associated homogeneous equation.

(2) Use the method of undetermined coefficients to find a particular solution of the given equation.

(3) Write down the general solution of the given equation.

(4) Find the solution satisfying the given initial conditions.

$$y'' - 6y' - 7y = -9e^{-2t}, \quad y(0) = 6, \quad y'(0) = 3$$

Q4

25 Points

In this question you will solve the following initial value problem in three steps:

(i) $y'' + y' - 2y = 2t$

(ii) $y(0) = 0, y'(0) = 1$

Step 1: Find the general solution of the associated homogeneous linear differential equation

$y'' + y' - 2y = 0.$

Step 2: Find one particular solution to the following inhomogeneous linear differential equation:

$y'' + y' - 2y = 2t.$

Hint: Use the method of undetermined coefficients to guess an appropriate trial solution $y_p(t)$.

Step 3: Use the answers you got in Steps 1 and 2 to find he solution to the original initial value problem:

(i) $y'' + y' - 2y = 2t$

(ii) $y(0) = 0, y'(0) = 1$