University of California, Los Angeles
Spring 2022

Instructor: C. Wang
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MATH 33B: Differential Equations
Midterm Exam 1

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* Please grade #1 ! #3 :)

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Q1

25 Points

Check whether the following differential form is closed and exact.

$(2t + 5y)dt + (5t - 6y)dy$

$\frac{\partial}{\partial y}(2t+5y) = 5$

$\frac{\partial}{\partial t}(5t-6y) = 5$

Since $\frac{\partial}{\partial y}(2t+5y) = \frac{\partial}{\partial t}(5t-6y) = 5$, the differential form is closed, and since $P(t) = 2t+5y$ and $Q(t) = 5t-6$ is continuous and differentiable, the differential form is also exact. over $\mathbb{R}$

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Q2
25 Points

A tank contains 100 gallons of brine made by dissolving 80 lb of salt in water. Pure water runs into the tank at the rate of 4 gallons/minute, and the mixture, which is kept uniform by stirring, runs out at the same rate. Find the amount of salt in the tank at any time t. Find the concentration of salt in the tank at any time t.

Rate in - Rate out

↑ no salt entering

↘ salt exiting

$$\frac{dy}{dt} = -\frac{y}{100} \cdot 4, \quad y(0) = 80$$

$$\frac{dy}{dt} = -\frac{y}{25}$$

$$\int \frac{dy}{y} = -\int \frac{1}{25}\,dt$$

$$\ln|y| = -\frac{t}{25} + C \rightarrow y = Ce^{-\frac{t}{25}}$$

$$80 = Ce^{0} \rightarrow C = 80 \rightarrow \boxed{y = 80e^{-\frac{t}{25}}}$$

Amount of salt at any time t

Concentration of Salt at any time t:

$$y = \frac{80e^{-\frac{t}{25}}}{100}$$

$$\boxed{y = \frac{4}{5}e^{-\frac{t}{25}}}$$

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Q3

25 Points

Solving the following separable differential equation (you need to give general solution in explicit form):

$dy/dt = ty$

$$\frac{dy}{dt} = ty \rightarrow \int \frac{dy}{y} = \int t\,dt$$

$$\ln|y| = \frac{t^2}{2} + C$$

$$|y| = e^{\frac{t^2}{2}+C} = e^C e^{\frac{t^2}{2}}$$

$$\boxed{y = Ce^{t^2/2}}$$

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~~Q4~~ ← Don't grade this one

~~25 Points~~

Solving the following initial value problem (no need to give the interval of existence):

$y'(t) + \frac{y(t)}{1-t} = 0, y(1) = 1.$

$\mu = e^{\int \frac{1}{1-t} dt} = e^{-\ln|1-t|} = e^{\ln(|1-t|^{-1})} = \frac{1}{|1-t|} \to \frac{1}{1-t}$

$\int \left(y \frac{1}{1-t}\right)' dt = \int 0 \, dt$

$\frac{y}{1-t} = C$

$y = C(1-t)$

$1 = C(1-1)$