22S-MATH-33B-LEC-3 Midterm Exam one

AARYAN DIVATE

TOTAL POINTS

100 / 200

QUESTION 1

1 50 / 50

✓ **- 0 pts Correct**

- 1 pts Click here to replace this description.

- 50 pts Click here to replace this description.

- 2 pts Click here to replace this description.

- 3 pts Click here to replace this description.

- 4 pts Click here to replace this description.

- 25 pts Click here to replace this description.

QUESTION 2

2 50 / 50

✓ **+ 50 pts Correct: if $$s(t)$$ is the weight of salt at time $$t$$, then $$s(t) = 80 e^{-t/25}$$, and the concentration is $$c(t) = \frac{4}{5} e^{-t/25}$$**

+ 0 pts Problem not selected

+ 15 pts $$s'(t) = \left(\textrm{rate in} \right) - \left(\textrm{rate out} \right)$$

$$=0 - 4\cdot \frac{s(t)}{100}$$

+ 20 pts Get the general solution

$$s(t) = C \cdot e^{-t/25}$$

+ 15 pts Use the initial condition

$$80= s(0) = C$$ to get the particular solution

$$s(t) = 80 e^{-t/25},$$ so $$c(t) = \frac{4}{5} e^{-t/25}$$

QUESTION 3

3 0 / 50

- 0 pts Correct

✓ **- 50 pts (Did not select Question 3)**

- 20 pts Incorrect placement of c

- 30 pts Error

- 40 pts Unsolved

- 15 pts Correct solution but did not find explicit form (i.e. solve for y)

- 25 pts (shifting so the total points are out of 100)

QUESTION 4

4 0 / 50

+ 15 pts Correct separation of variables and/or integrating factor.

+ 15 pts Correct integrals.

+ 10 pts Correctly solved for y (general solution).

+ 10 pts Correctly used the initial condition to find the particular solution.

✓ **+ 0 pts Did not choose this problem.**

+ 25 pts Click here to replace this description.

University of California, Los Angeles
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Instructor: C. Wang
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MATH 33B: Differential Equations
Midterm Exam 1

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Evaluate #1 and #2

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Q1

25 Points

Check whether the following differential form is closed and exact.

$(2t + 5y)dt + (5t - 6y)dy$

in form, $P\,dt + Q\,dy$,

is closed if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial t}$

$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(2t+5y) = 5$

$\frac{\partial Q}{\partial t} = \frac{\partial}{\partial t}(5t-6y) = 5$

$5 = 5$, so therefore $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial t}$, meaning this differential form is closed. Since t and y are continuous and differentiable for all $\mathbb{R}$ and the form is closed, it is also exact.

So, the differential form is closed and exact.

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Q2

25 Points

A tank contains 100 gallons of brine made by dissolving 80 lb of salt in water. Pure water runs into the tank at the rate of 4 gallons/minute, and the mixture, which is kept uniform by stirring, runs out at the same rate. Find the amount of salt in the tank at any time t. Find the concentration of salt in the tank at any time t.

note: amount of salt in tank has units of lbs; concentration of salt in tank has units $\frac{lbs}{gal}$.

pure water: $\frac{0\,lbs}{gal}$

rate in: $\frac{0\,lbs}{gal} \cdot \frac{4\,gal}{min} = \frac{0\,lbs}{min}$

rate out: $\frac{4\,gal}{min} \cdot \frac{y(t)}{100\,gal} = \frac{4y(t)}{100} = \frac{y(t)\,lbs}{25\,min}$ ← concentration in tank

let $y(t)$ represent amount of salt in tank at time t

$y'(t) = \text{rate in} - \text{rate out}$

$= 0 - \frac{y(t)}{25}$

so $y'(t) = -\frac{y(t)}{25}$, initial condition $y(0) = 80$

$y'(t) + \frac{y(t)}{25} = 0 \rightarrow 0$ is $g(t)$, $\frac{1}{25}$ is $p(t)$

$\mu(t) = \exp\left(\int \frac{1}{25}\,dt\right)$

$= e^{t/25}$

so $y(t) = \frac{1}{\mu(t)}\left(\int \mu(t)g(t) + C\right)$

$y(t) = e^{-t/25}\left(\int 0 + C\right)$

$y(t) = Ce^{-t/25}$

$y(0) = 80 = Ce^{0}$

$80 = C$

so $\boxed{y(t) = 80e^{-t/25}}$ ← amount of salt in tank at time t

Concentration of salt in tank: $\frac{y(t)}{100\,gal} = \frac{80e^{-t/25}}{100} = \boxed{\frac{4}{5}e^{-t/25}}$

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Q3

25 Points

Solving the following separable differential equation (you need to give general solution in explicit form):

$dy/dt = ty$

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Q4

25 Points

Solving the following initial value problem (no need to give the interval of existence):

$y'(t) + \frac{y(t)}{t} = 0, y(1) = 1.$