University of California, Los Angeles
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Instructor: C. Wang
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MATH 33B: Differential Equations
Midterm Exam 1

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Q1

25 Points

Check whether the following differential form is closed and exact.

$(2t + 3y)dt + (3t - 6y)dy$

$P(t,y) = 2t + 3y$

$Q(t,y) = 3t - 6y$

$\frac{\partial P}{\partial y} = 3$

$\frac{\partial Q}{\partial t} = 3$

Continuous and differentiable over $\mathbb{R}, \mathbb{R}$

$\therefore$ it is closed and exact

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Q2

25 Points

Solving the following initial value problem (no need to give the interval of existence):

$y' + (2/t)y = sin(t)/t^2, y(\pi/2) = 2/\pi$

$y' + \frac{2}{t}y = \frac{\sin t}{t^2}$ $\quad \mu(t) = e^{\int \frac{2}{t}dt} = e^{2\ln(t)} = t^2$

$t^2\left(y' + \frac{2}{t}y = \frac{\sin t}{t^2}\right)$

$t^2y' + 2ty = \sin(t)$

$\int (t^2y)' = \int \sin(t)$

$t^2y = -\cos(t) + C$

$y = \frac{-\cos(t)}{t^2} + \frac{C}{t^2}$ $\quad y(\pi/2) = \frac{2}{\pi}$

$\cos(\frac{\pi}{2}) = 0$

$\frac{2}{\pi} = \frac{C}{(\pi/2)^2}$

$C = \pi/2$

$$y = -\frac{\cos(t)}{t^2} + \frac{\pi/2}{t^2}$$

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Q3

25 Points

Solving the following separable differential equation (only need to give general solution in implicit form):

$dy/dt = ty$

$\frac{dy}{dt} = ty$

$\int \frac{1}{y} dy = \int t\, dt$

$\ln|y| = \frac{t^2}{2} + C$

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Q4

25 Points

A tank contains 100 gallons of brine made by dissolving 80 lb of salt in water. Pure water runs into the tank at the rate of 4 gallons/minute, and the mixture, which is kept uniform by stirring, runs out at the same rate. Find the amount of salt in the tank at any time t. Find the concentration of salt in the tank at any time t.

Amount of salt = $x(t)$

Concentration of salt = $y(t)$

$x(0) = 80$

$\frac{dx}{dt} = 0.4 - \left(\frac{x}{100}\right) \cdot 4$

$\frac{dx}{dt} = -\frac{x}{25}$

$\int \frac{dx}{x} = \int -\frac{1}{25}\,dt$

$\ln x = -\frac{t}{25} + C$

$x = Ce^{-t/25}$ $\quad x(0) = 80$

$80 = C \cdot e^0 = C$

$\boxed{x(t) = 80 \cdot e^{-t/25}}$

$y = \frac{x}{100}$

$y(0) = 80/100 = 0.8$

$\frac{dy}{dt} = -\frac{y}{2500}$

$\int \frac{dy}{y} = \int -\frac{1}{2500}\,dt$

$\ln y = -\frac{t}{2500} + C$

$y = Ce^{-\frac{t}{2500}}$ $\quad y(0) = 0.8$

$0.8 = C$

$y(t) = 0.8 \cdot e^{-t/2500}$

$y = \frac{x}{100}$

$\boxed{y(t) = 0.8 \cdot e^{-t/25}}$