21F-MATH33B-1 Midterm exam 1

LEONARD CHEN

TOTAL POINTS

100 / 100

QUESTION 1

1 Q1 25 / 25

✓ **- 0 pts Correct**

- 20 pts Click here to replace this description.

- 5 pts Click here to replace this description.

- 2 pts Click here to replace this description.

- 1 pts Click here to replace this description.

- 6 pts Click here to replace this description.

- 4 pts Click here to replace this description.

QUESTION 2

2 Q2 25 / 25

✓ **- 0 pts Correct**

- 7 pts Incorrect integrating factor

- 7 pts Incorrect general solution for the differential equation

- 7 pts Incorrect solution for the initial value problem

- 2 pts Computational errors

QUESTION 3

3 Q3 25 / 25

✓ **+ 10 pts Correctly separated variables.**

✓ **+ 5 pts Correctly integrated at least one side of the equation.**

✓ **+ 5 pts Correctly integrated both sides of the equation.**

✓ **+ 5 pts Correct final answer in some (implicit or explicit) form (must include the +C).**

+ 0 pts Incorrect/No Submission

QUESTION 4

4 Q4 25 / 25

✓ **- 0 pts Correct**

- 5 pts Incorrect differential equation for the amount of salt.

- 10 pts Incorrect general solution to the differential equation for the amount of salt.

- 5 pts Incorrect particular solution to the differential equation for the amount of salt.

- 5 pts Incorrect function for the concentration of salt.

- 2 pts Computational errors.

University of California, Los Angeles
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Instructor: C. Wang
Date: Oct. 20, 2021

MATH 33B: Differential Equations
Midterm Exam 1

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Q1

25 Points

Check whether the following differential form is closed and exact.

$(2t + 3y)dt + (3t - 6y)dy$

$\frac{\partial}{\partial y}(2t+3y) = 3$

$\frac{\partial}{\partial t}(3t-6y) = 3$

As both $2t+3y$, $3t-6y$ are defined over the rectangle of all real numbers, and the cross-partials are equal, the differential form is both closed and exact.

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Q2

25 Points

Solving the following initial value problem (no need to give the interval of existence):

$y' + (2/t)y = sin(t)/t^2, y(\pi/2) = 2/\pi$

$$\mu(t) = e^{\int \frac{2}{t} dt} = e^{2\ln|t|} = |t|^2 = t^2$$

$$t^2\left(y' + \frac{2}{t}y\right) = \sin(t)$$

$$(t^2 y)' = \sin t$$

$$\int (t^2 y)' = \int \sin t \, dt$$

$$t^2 y = -\cos t + C$$

$$y(t) = \frac{-\cos t + C}{t^2}$$

$$\frac{2}{\pi} = \frac{-\cos\left(\frac{\pi}{2}\right) + C}{\left(\frac{\pi}{2}\right)^2}$$

$$\frac{\pi}{2} = 0 + C, \quad C = \frac{\pi}{2}$$

$$\Rightarrow y(t) = \frac{-\cos t + \pi/2}{t^2}$$

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Q3

25 Points

Solving the following separable differential equation (only need to give general solution in implicit form):

$dy/dt = ty$

$$\frac{dy}{y} = t\,dt$$

$$\int \frac{dy}{y} - \int t\,dt = 0$$

Implicit: $\ln|y| - \frac{1}{2}t^2 + C = 0$

Explicit: $\ln|y| = \frac{1}{2}t^2 + C$

$$|y| = e^{\left(\frac{1}{2}t^2 + C\right)}$$

$$y = \pm C e^{\frac{1}{2}t^2}$$

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Q4

25 Points

A tank contains 100 gallons of brine made by dissolving 80 lb of salt in water. Pure water runs into the tank at the rate of 4 gallons/minute, and the mixture, which is kept uniform by stirring, runs out at the same rate. Find the amount of salt in the tank at any time t. Find the concentration of salt in the tank at any time t.

The volume is constant, $s(t)$ is salt in tank at time t

$c(t)$ is concentration at time t

$s'(t) =$ rate in $-$ rate out

$$s'(t) = 0 - \frac{4}{100} \cdot s(t)$$

$$s'(t) = -\frac{1}{25} s(t)$$

$$\frac{ds}{dt} = -\frac{s(t)}{25}$$

$$\int \frac{ds}{s(t)} = -\int \frac{dt}{25}$$

$$\ln|s(t)| = -\frac{1}{25}t + C$$

$$s(t) = e^{-\frac{t}{25}+C} = Ce^{-\frac{1}{25}t}$$

At $t=0$, $s(0)=80$, so $80 = Ce^0$, $C = 80$

Thus $\boxed{s(t) = 80e^{-\frac{1}{25}t}}$

the concentration is just $\frac{s(t)}{V}$,

so $c(t) = \frac{80}{100} e^{-\frac{1}{25}t}$

$$c(t) = \boxed{\frac{4}{5} e^{-\frac{1}{25}t}}$$