

Q1

10 Points

[New instruction: on *all* of these problems (Questions 1-5) you may use a calculator/compute for *arithmetic only*, i.e., simplifying expression which involve $+$, \cdot , $-$, $/$.]

Answer the following multiple-choice questions. Double-check your answers, since no partial score would be given for these questions.

Q1.1

2 Points

How many solutions does the following system have:

$$x + 2y + 3z = 1$$

$$x + 3y + 4z = 3$$

$$x + 4y + 5z = 4$$

- one
- infinitely many
- none

Q1.2

2 Points

The partial fraction decomposition of $\frac{t^2 - 2}{(t + 1)^2(t^2 + 1)}$ should be of the form

- $\frac{At}{(t+1)^2} + \frac{Bt+C}{t^2+1}$
- $\frac{A}{(t+1)} + \frac{B}{(t+1)^2} + \frac{C}{t^2+1}$
- $\frac{A}{(t+1)} + \frac{B}{(t+1)^2} + \frac{Ct+D}{t^2+1}$
- $\frac{A}{(t+1)} + \frac{B}{(t+1)^2} + \frac{Ct}{t^2+1}$

Q1.3

2 Points

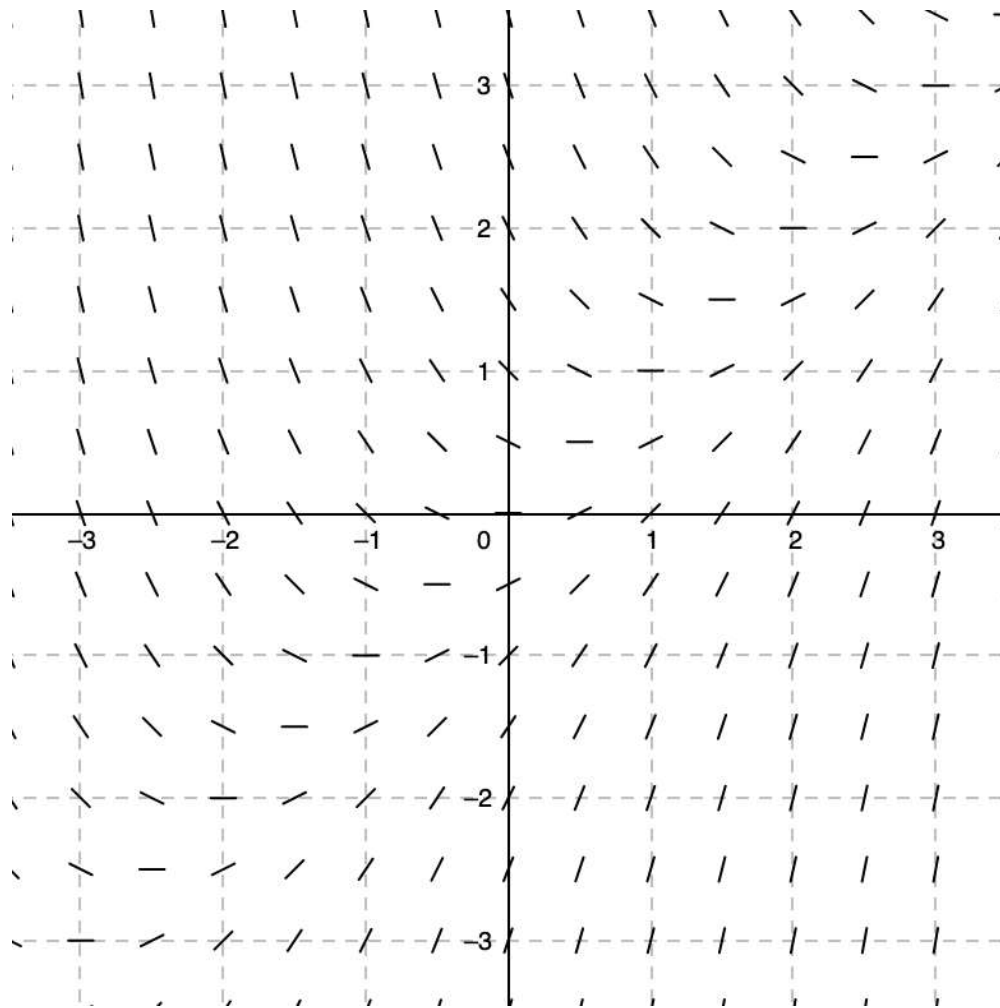
Match the following differential equations with their direction fields.

A: $y' = 2 - y$

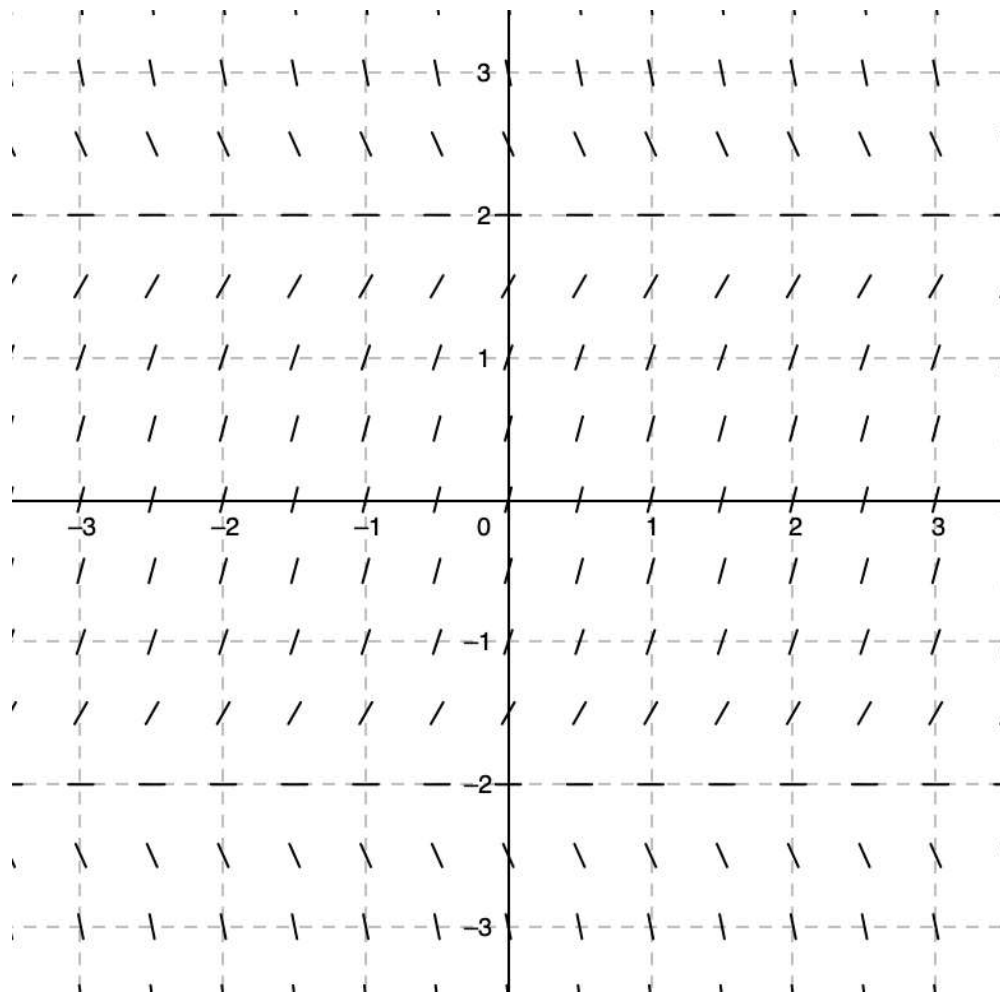
B: $y' = (y + 2)(2 - y)$.

C: $y' = t - y$

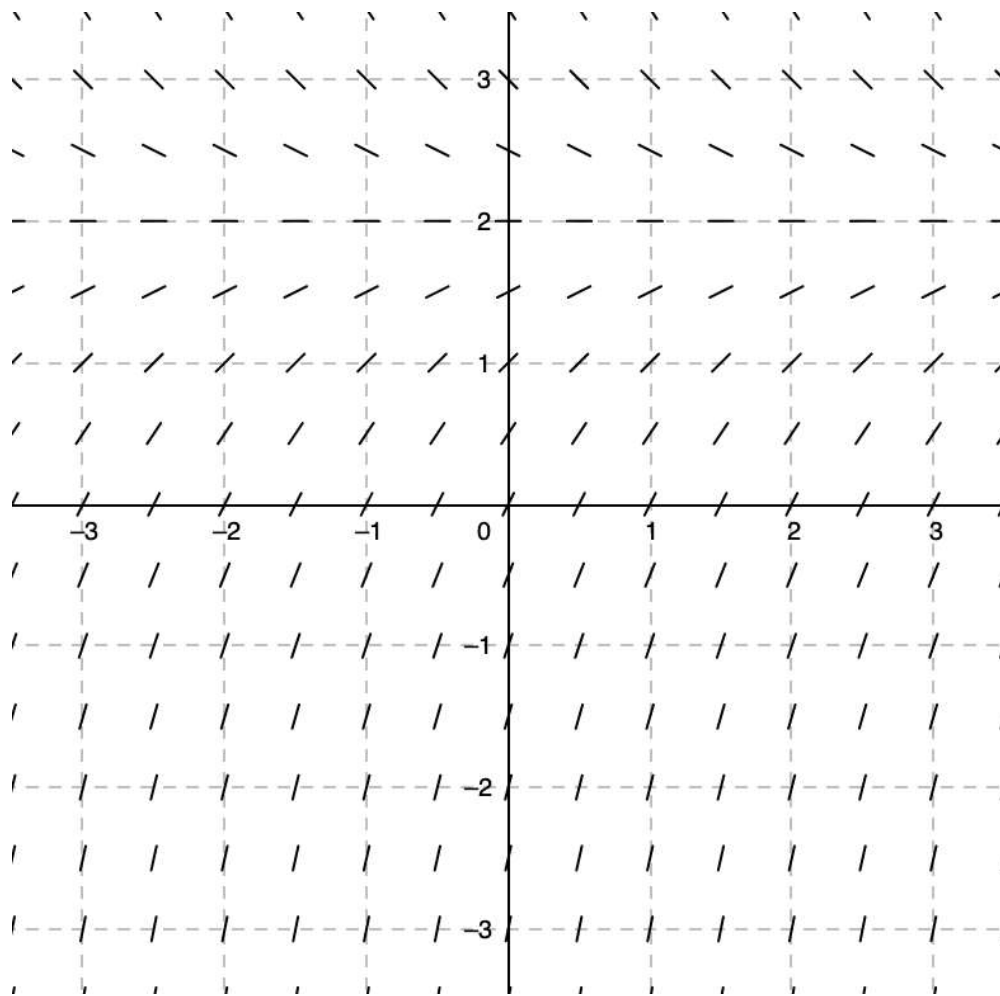
(1)



(2)



(3)



(1) is for

C

(2) is for

B

(3) is for

A

Q1.4

2 Points

Which of the following equations can be rearranged into separable equations?

$y' - y^2 = t$

$y' + yt^2 = t^2$

$ty' - e^t \sin(y) = e^t$

$y' - \sin(t)y = t$

Q1.5

2 Points

Which of the following differential forms are exact?

$(y^2 - t^2) dt + (2ty - y) dy$

$ye^t dt - e^t dy$

$3 \cos(3t - y) dt - \cos(3t - y) dy$

$(2t^2y - 1) dt + t^3 dy$

Q2

20 Points

Find the general solution for the following differential equation:

$$y' + ay = t^n e^{-at}, \text{ where } a \in \mathbb{R} \text{ and } n \in \mathbb{N}.$$

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Q3

25 Points

A tank contains 100 gallons of brine made by dissolving 80 lb of salt in water. Pure water runs into the tank at the rate of 4 gallons/minute, and the mixture, which is kept uniform by stirring, runs out at the same rate. Find the amount of salt in the tank at any time t . Find the concentration of salt in the tank at any time t .

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**Q4**

20 Points

Solve the following initial value problem and determine the respective interval of existence:

$$te^{t^2} + yy' = 0, \quad y(0) = 1.$$

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**Q5**

25 Points

Check that the following differential form are exact and find the solution to the corresponding initial value problem:

$$\frac{y}{t+1} dt + (\ln(t+1) + 3y^2) dy = 0, \quad y(0) = 1.$$

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Midterm1



STUDENT

DAVID XIONG

TOTAL POINTS

98 / 100 pts

QUESTION 1

(no title)

8 / 10 pts

1.1 (no title)

2 / 2 pts

1.2 (no title)

2 / 2 pts

1.3 (no title)

2 / 2 pts

1.4 (no title)

0 / 2 pts

1.5 (no title)

2 / 2 pts

QUESTION 2

(no title)

20 / 20 pts

QUESTION 3

(no title)

25 / 25 pts

QUESTION 4

(no title)

20 / 20 pts

QUESTION 5

(no title)

25 / 25 pts